A Refutation of Arrow's Theorem

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To my Mother

and

In memory of my Father

Contents

Introduction

Arrow's theorem is the most celebrated result in social choice theory. According to the philosopher Robert Paul Wolff, what Arrow has shown is that inconsistency "infects virtually every method of social choice which can lay a reasonable claim to being called 'democratic' "(1970 63). In 1972, the economist Paul Samuelson described the implications of Arrow's theorem as follows:

> The search of the great minds of recorded history for the perfect democracy, it turns out, is the search for a chimera, for a logical self–contradiction. . . . Now scholars all over the world –– in mathematics, politics, philosophy, and economics –– are engaged in trying to salvage what can be salvaged from Arrow's devastating discovery that is to mathematical politics what Gödel's 1931 impossibility–of–proving–consistency theorem is to mathematical logic. (1966 4:938; see also 4:935–6 and 3:409–12; also quoted in Gardner 1974 120)

Although Arrow's "impossibility theorem" is still generally accepted as correct and important, it has its critics.[1] The most common criticisms are that the result is correct but irrelevant, or that one or another of Arrow's assumptions is wrong. Yet the theorem has not been the object of a full–scale attack. That is what will be attempted here. There is no claim that all the objections raised are new. They are not. But I believe that, taken together, they damage the proof beyond repair. My intention is to refute not only the theorem but, in addition, to show that its philosophical basis is antidemocratic and should be abandoned. Our age is one in which democratic revolutions are occurring in many places in the world. It is important, I think, to show that there is no justification for the claim that

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[1] For a bibliography on social choice, see Sen 1987 390–3. Arrow's views on social choice can most conveniently be found in Arrow 1963 and in Arrow 1983. The first volume of the latter entitled *Social Choice and Justice* contains most of the articles relevant to the impossibility theorem. Throughout this monograph I limit my criticisms to Arrow but the critical comments, if correct, have a wider application.

democracy is flawed, even if that claim be made only in a purely logical sense.

I have tried to make this refutation as simple and clear as possible. It is no more technically difficult than the original proof. Nor does it presuppose familiarity with that proof. It may thus be read first although there is no substitute for going back to Arrow's writings on the subject. The book is divided into nine chapters. Chapter 1 is mainly concerned with the role of random procedures in democracy. Chapters 2 through 5 will show the unreasonableness of Arrow's four principal premisses: Collective Rationality, Pareto Principle, Independence of Irrelevant Alternatives, and Nondictatorship. Chapter 6 focuses on the validity of the argument. It attempts to show that there are nontrivial errors in Arrow's argument from the premisses to the theorem. Chapter 7 gives an analysis of what would constitute a good impossibility proof for democratic social choice. It tries to show that Arrow greatly underestimated its difficulty. Chapter 8 criticizes the philosophical presuppositions of the proof and endeavors to show that they are contrary to the fundamental assumptions of democracy. Chapter 9 claims that the widespread acceptance of those presuppositions has been a pernicious influence on social choice theory. It explores the question of what social choice theory might have become without that influence.

"No one pretends that democracy is perfect or all–wise. Indeed, it has been said that democracy is the worst form of Government except all those other forms that have been tried from time to time . . . "(1974 7:7566). These famous words of Winston Churchill are in effect challenged by Arrow's theorem which suggests that, from a purely logical point of view, dictatorship is superior to democracy since the former can be consistent whereas the latter can not be. To reach his conclusion Arrow presupposes a conception of democracy which I believe to be both unhistorical and untenable. In these pages I take Athenian and American democracies to be the paradigms of democracy, contrary to Arrow's conception. An explication of my conception of democracy is offered as far as is necessary for the purposes of a refutation. An extensive defense can be found in my forthcoming book called *The Court of Common Reason.* In fact, this essay was for some time an appendix to that book. It began, however, as a lecture I gave over thirteen years ago. Since then it has been revised and enlarged many times. I have corrected not only outright errors but other statements I have come to believe were quite ill–advised. That this version is much improved is due to the assistance and comments of numerous people. For their help with, or criticisms of, earlier drafts I would like to thank Alfred Covello, Laura Foley, John Georges, John Gillroy,

Andrew Gold, Richard Lee, Dan Lloyd, Alfred MacKay, Anthony Macro, Joel Marks, Judith Moran, William Puka, Vincent Smith, Peter Suber, Timothy Teeter, Stephen Utz, and Maurice Wade. In addition, Peter Skagestad gave exceptionally detailed comments which saved me from some serious errors and Miller Brown not only encouraged me every step of the way, he gave the penultimate version of the essay a wonderfully critical reading. I also benefitted from a discussion of Arrow's theorem with William Fairchild, Christopher Leary, and Alan Taylor. Of course, the faults that remain are entirely my responsibility. Finally, I am indebted to the Mellon, Dana, and Sloan Foundations for providing the funds which enabled me to carry out this research and to Trinity College (Hartford) because it was through Trinity that I became the beneficiary of those funds.

1

Democracy and Random Procedures

Arrow begins by defining an *environment* to be the set of available actions open to an individual.[2] By finding out what an individual would choose from many different environments we can determine that individual's value system and, thereby, determine that when the individual is faced with two alternative actions, he prefers one to the other, or is indifferent. This enables us to ascertain the individual's preference orderings for any environment, and a set of such orderings for the members of society. "The fundamental problem of public value formation . . . ," Arrow states, "is the construction of constitutions," where "a constitution is a rule that assigns to any set of individual preference orderings a rule for making society's choices among alternative social actions in any possible environment"(1967 14, 15).

Arrow quite rightly emphasizes that there be "reasonable" conditions placed on democratic constitutions. He proposes four (1983 1:70–1):

> *Collective Rationality*: For any given set of orderings, the social choice function is derivable from an ordering.
>
> *Pareto Principle*: If alternative x is preferred to alternative y by every single individual according to his ordering, then the social ordering also ranks x above y.
>
> *Independence of Irrelevant Alternatives*: The social choice made from any environment depends only on the orderings of individuals with respect to the alternatives in that environment.
>
> *Nondictatorship*: There is no individual whose preferences are

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[2] Arrow has published numerous versions of the proof of his theorem. In the subsequent exposition and criticism I usually quote from the oldest place where a particular idea is expressed unless there is a specific reason (such as consistency of exposition in this monograph) which would dictate otherwise. The criticisms I wish to make generally apply to all the versions, although for some criticisms, changes in exposition would be needed.

automatically society's preferences independent of the preferences of all other individuals.

From these assumptions Arrow derives his theorem which states that it is impossible to satisfy these four conditions simultaneously. The theorem Arrow finds "quite embarrassing," apparently because it shows that there is an inherent contradiction within democratic theory.

We will begin our critique by considering the use of random techniques in democratic social choice. Arrow ignores these techniques even though their use is very old and they continue to be part of democratic practice. In ancient Greece it is probable that the use of random techniques for decisions originated in religion. A hint of this origin is given in Plato's *Laws* where it is stated that "because of the discontent of the many they are compelled to make use of the equality of the lot, but when they do, they should pray both to the god and to good luck to correct the lot in the direction of what is most just"(1980 144 [757E]).[3] Nevertheless in Athens the use of the lot -- that is, any of a variety of random devices -- for the choice of magistrates became quite secular at least as early as 487 B. C. (Larsen 1949 171). Early on, if not from the beginning, the lot was associated with democracy. Plato, for example, says that in a democracy "most of the offices [are] assigned by lot"(1985 248 [557A]) and Aristotle states that such election is one of the characteristics of democracy(1984 2:2091 [1317b 20–2]).

In ancient Rome the use of lot was also "not insignificant"(Staveley 1972 230). In particular, Romans used it to break tie votes (234 and Taylor 1966 80; see also Cicero 1935 477 [*Pro Cnaeo Plancio* xxii 53]). This practice continues today in parliamentary procedure. For example, *Demeter's Manual of Parliamentary Law and Practice* states that ". . . in the event of a tie vote for a final choice, the candidates . . . shall decide the result by lot, or by coin"(1969 246). The practice is also widely used in many general elections. A check of the current election codes of the fifty states shows that at least three–quarters use a random technique to break tie votes for at least some elections. Nor are its uses always

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[3] The ancient Hebrews also used random choice to indicate God's intentions. An example may be found in the election of Saul. "Then Samuel brought all the tribes of Israel near, and the tribe of Benjamin was taken by lot. He brought the tribe of Benjamin near by its families, and the family of the Matrites was taken by lot; finally he brought the family of the Matrites near man by man and Saul the son of Kish was taken by lot." Later Samuel asks: "Do you see him whom the Lord has chosen?"(*1 Samuel* 10:20–24).

limited to minor offices. For example, Utah, among others, uses lot to break ties for presidential electors (UEL 113 [20–1–9]), a procedure which makes it is theoretically possible that lot will someday determine who becomes President.

Pennsylvania provides another example of the potential importance of the use of random techniques. On November 10, 1978, the Democrats and Republicans each controlled 101 seats in state House of Representatives (as a result of the election earlier in the week). For the remaining seat the Democrat and the Republican candidates each received 8551 votes. As *The New York Times* reported the next day: "If the tie persists after the recount, the state Election Code provides for a lottery in which . . . each of the candidates would draw a number from a bag. The one who picked the lowest number would win the seat"(TVRO 1978 8). Ties are not infrequent with respect to less important offices. In various elections in Ohio during the past decade, for example, a tie vote being decided by lot has on average been occurring more than once a year.[4] The use of lot to break ties is thus both old and well–entrenched in democratic practice. There is even a ruling by the Supreme Court of Indiana which states that the requirement of the Indiana constitution that "all elections shall be by ballot" is not violated by using lot to break ties since that use "deprives no elector of his vote, and gives to each vote its full force and weight"(JVS 1891 422).[5]

Nor are random procedures used only for breaking ties in contemporary democratic practice. In various states random procedures determine the order of candidates on the ballot (California), the placement of party columns on the ballot (Idaho), the sample of names on petitions to be checked for validity (Florida), the selection of jurors (Connecticut), the order of simultaneously filed petitions (Illinois), the choice of which judge is to preside over a particular trial (New York), and the issuance of hunting permits (Minnesota). On a national level, the determination of which of the first two senators from Alaska should be senior was

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[4] I am grateful to Margaret Rosenfield, the Ohio Director of Elections Programs, for providing me with this information.

[5] I have no intention of suggesting that ties in all circumstances can only be properly broken by use of random devices. Other means are certainly used such as letting the decision be made by the legislature, the governor, a judge, the people (via a runoff election) or the candidates (that is, they can decide the decision method). The question of the proper means of handling a tie is an old one in American government as is shown by the contested election of Maryland's Jeremiah Cosden to the U. S. House in 1820. See "Reed vs. Cosden" in Lowrie and Franklin 1834 and JHRUS 1821.

settled by the toss of a silver dollar, after which another toss resolved who should first draw lots to decide the length of the terms (BWTSS 1959; Trussell 1959).[6] And, of course, a lottery is used in the choice of draftees.

There are numerous reasons why random techniques are employed in democracies. They include: promoting freedom, thwarting corruption, enlarging the pool of those who actually participate in governance, avoiding (compulsory) taxation when additional revenue is needed, undermining political intrigue,[7] supplying an unbiased means of settling ties, furnishing a device to settle disputes which might otherwise go on interminably,[8] and providing a decision procedure when every other method threatens democratic equality (as in issuing hunting permits or in drafting young men). "What is distinctive about the democratic society," Arrow correctly notes, "is that mutual relations are conceived of as symmetric in spirit. Individuals, at least in some initial sense, have to be treated equally by the society, if it is to be worthy of the name 'democratic'"(1974 23). A lottery which is used to determine draftees or hunting permits is an attempt to achieve such equality and therefore its use is not arbitrary precisely because of its arbitrary outcome.

Just as random procedures can be employed to break a symmetrical deadlock (that is, a tie) in the case of an even number of voters, they can likewise be used to decide a symmetrical deadlock involving an odd number of voters. The paradox of voting can illustrate such a deadlock. Arrow describes the paradox as follows: "There are three alternatives, x, y, and z, among which choice is to be made. One–third of the voters have the ranking x, y, z; one–third, the ranking y, z, x; and one–third, the ranking z, x, y." So far so good. However, Arrow goes on to say: "Then a majority of the voters prefer x to y, a majority prefer y to z, and a majority prefer z to x"(1985 143). This conclusion, however, is not

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[6] The same procedure was also used for Hawaii (see Trussell 1959a 1,15).

[7] "In democratic states legislation ought to provide for appointment by lot to the less important and the majority of the offices (for thus faction will be avoided). . ."(Aristotle 2:2276 [1424a 13–4]). (The quotation is from *Rhetoric to Alexander* which, though part of the traditional corpus of writings of Aristotle, is almost certainly not by him.)

[8] Avoiding prolonged and unfruitful dissension is the reason that members of the U. S. House of Representatives of equal seniority draw lots to determine their office suites. The use of random techniques for this reason is quite ancient: "The lot puts an end to disputes and decides between powerful contenders"(*Proverbs* 18:18).

paradoxical in any important sense. Let us see why. Imagine someone gave the following argument in an attempt to give the paradox of voting for two alternatives: "Suppose we have six voters consisting of three conservatives and three liberals. The three conservatives prefer *u* over *v*, and three liberals prefer *v* over *u*. Hence half prefer *u* to *v* and half prefer *v* to *u*. There is thus an inconsistency. (Furthermore, each individual is decisive but the whole is indecisive and therefore we have another inconsistency.)" I doubt anyone would be impressed by this argument. First, there is an equivocation on "half" which gives the argument its air of paradox. If one says "hence three conservatives prefer *u* to *v* and three liberals prefer *v* to *u*" there is little temptation to conclude that there is a paradox. The "paradox of voting for two alternatives" is just a two–way tie for first place and a two–way tie for second place. (And neither is the decisiveness of each half, or of each individual, a guarantee of the decisiveness of the whole.) The paradox of voting is similar in that it is a three–way tie for first place, a three–way tie for second place, and a three–way tie for third place. The term "majority" is similarly equivocal (referring to three different groups) and the air of paradox depends on a fallacy of equivocation. (The decisiveness of each third, or each individual, is again no guarantee of the decisiveness of the whole.) Since the paradox of voting is a three–way tie for each place, a lottery provides a simple, fair and democratic means of social choice. For example, if the three alternatives are political candidates, random techniques promote democratic equality in the sense that each has an equal chance of gaining first, second, or third place.[9]

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[9] Again, I do not want to suggest that random techniques are the best means in every conceivable situation. For example, in ancient Athens when a jury was tied between two penalties the lesser penalty was chosen. Nevertheless, in the abstract, without any other factor being considered, random devices are perfectly adequate. It is possible to create more complicated versions of the paradox of voting –– that is, a situation with cyclical majorities –– which do not involve ties. In my judgment these are best treated by approval voting. In this scheme voters choose from a list of candidates or options all those of which they approve. For example, a voter could choose a third–party candidate (with little chance of winning) *and*, say, the Democrat. Then the winner is the one with the most votes, that is, the one which has the approval of the most voters. (See Brams and Fishburn 1983 for a complete explication of approval voting.) Ties can be broken by lot or some other appropriate means. Since Arrow concentrates his discussion on the paradox of voting with a symmetrical deadlock this monograph will as well. However, it should be noted that he probably rejects approval voting as a way around the difficulty since as late as 1985 he states that the paradox of voting "is not due to a removable imperfection in the method of majority voting"(1985 143). This is eight years after the first articles on the approval voting and two years after the standard work of Brams and Fishburn.

Notice that random procedures can also be used to decide questions of order and thus undermine the power of political leaders to manipulate outcomes of voting. For example, if balloting allows only the expression of preferences between two choices then, in the paradox of voting, if there is an initial contest between x and y, x would win but subsequently lose to z in the run–off. If the initial contest is between x and z, z would win but then lose to y. Finally, if the initial contest is between y and z, y would win but then lose to x. In such a situation the person who decides the order of voting would decide the outcome. In this "voting order" version of the paradox the use of a random technique is again an appropriate means for determining the order of voting and thus the winner. The use of lot to determine the order of voting is very old. Thus the ancient Romans used it to determine the order in which tribes voted (and thus the order in which results were announced) because "of the significant effect which the order of announcing results could have upon the outcome of an election"(Staveley 1972 180; see also Taylor 1966 80 and Hall 1964 278).[10]

The paradox of voting as a symmetrical deadlock is a paradox not in a logical or philosophical sense but only in the colloquial sense which means "unexpected result." It was not expected that a majority of the voters could sometimes be against any choice; it was not expected that the order of voting can sometimes determine the outcome. Since in the abstract the symmetrical deadlock is so easily and democratically resolved, it is perhaps comparable to the paradox that one can be twenty–one having had only five birthdays.[11] This statement, although true, is troublesome only if one forgets about February 29th. Similarly, the symmetrical deadlock of the paradox of voting is baffling only if one forgets about the well–established random procedures of democratic theory and practice. From a logical point of view it bears analogy to the Aristotle's story of the man who, "though exceedingly hungry and thirsty, and both equally, yet being

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[10] Random techniques can also resolve other paradoxes. For example, consider what I shall call the paradox of the green. Imagine a community where the members are asked whether they want a rule against walking on the green. Excepting one person, half are for the rule, and half against it. The exception wants the rule if and only if most members don't want it. Should the social policy contain a rule against walking on the green? Flipping a coin provides an escape from this dilemma.

[11] I borrow this example from Quine who, in turn, borrowed it from Gilbert and Sullivan. See Quine 1966 3. However, Quine's use of the example is unrelated to Arrow's theorem.

equidistant from food and drink, is therefore bound to stay where he is"(1984 1:486 [296b 31–3]). Of course, the man is not bound to stay where he is. He may make the decision on the basis of other considerations, such as the fact that the drink, but not the food, is near a comfortable place to sit. A symmetrical deadlock puts the election official in the position of the man. Since the number of votes is equally divided she must decide on the basis of other considerations. However in most cases those considerations will undermine, or appear to undermine, democratic equality. Hence she resorts to a random procedure (which the man could naturally use as well). To deprive the democrat of random procedures is akin to depriving the Euclidean geometer of a Euclidean axiom. In setting up his system Arrow ignores democratic tie–breaking procedures and instead understands the paradox of voting in the logical or philosophical sense of an antinomy, an antinomy which requires a fundamental revision of the foundation of democratic theory. In fact, Arrow states that the second part of the proof of his theorem is "an appropriate adaption of the paradox of voting"(1963 100) and that the inconsistency of the four conditions on the social decision procedure "is in fact a generalized form of the paradox of voting"(1973 283). Elsewhere he claims that ". . . there is no method of voting which will remove the paradox of voting . . . neither plurality voting nor any scheme of proportional representation, no matter how complicated"(1950 342). Yet insofar as the paradox involves a symmetrical deadlock, random procedures -- such as those in Pennsylvania's Election Code -- would remove it democratically. And where a significant number of voters is involved, symmetrical deadlocks are highly unstable and recounts often will do away with them. In fact, a recount did away with Pennsylvania's deadlock (DRCPH 1978).[12]

In 1928 John von Neumann pointed out that, for certain games, the best strategy is to use no strategy in the usual sense but rather to depend on a random selector. He states: "The dependence on chance . . . is such an intrinsic part of the game itself . . . that there is no need to introduce it artificially by way of the rules of the game; even if the formal rules contain no trace of it, it will assert

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[12] New elections, it should be noted, are often impractical (since they may cost the state more than is available in the budget) and unfair (since they favor the candidate whose supporters are relatively wealthy). (See Herzberg 1986). Other means of resolving ties such as allowing the legislature or the governor to decide the issue are possible but, in theory, the legislature might itself be tied or the governor indecisive. In any case, Arrow's system lacks any means for dealing with ties so should we be surprised that it is incapable of dealing with the paradox of voting?

itself"(von Neumann 1959 26). The ancient Greeks had a similar insight with respect to the "game" of democracy. One of the purposes of the lot was, at certain times in the democratic process, to use no strategy in the usual sense but rather to depend on a random selector, in order that democracy could avoid defeat at the hands of the antidemocrats, who might otherwise successfully manipulate democratic procedures.[13] A golden opportunity for the antidemocrat -- or any faction -- is at the point of democratic indecision created by a symmetrical deadlock. Here, even if the formal rules of voting contain no trace of it, chance will assert itself.

Before closing this chapter it is worth noting that voting puzzles were known in ancient times. Gellius, for example, refers to a debate in which the topic was the following: "Seven judges are to hear the case of a defendant, and judgment is to be passed in accordance with the decision of a majority of their number. When the seven judges had heard the case, two decided that the defendant ought to be punished with exile; two, that he ought to be fined; the remaining three, that he should be put to death"(1927 2:207). This sort of example is also mentioned by Heliodorus (1961 13) and Fortunatianus (RLM 1964 97) and is the subject of declamations by Quintilian (1984 259–60) and Libanius (1903 526–43). Pliny the Younger gives a discussion of the actual case of the murder of Africanus Dexter, which had been discussed in the Senate. The issue was the disposition of the accused freedmen. Should it be acquittal or banishment or death? The Senate is divided so that no one of these possibilities commands a majority. In the course of his analysis Pliny states the following: "Let us suppose there had been only three judges appointed to hear this cause, one of whom was of the opinion that the freedmen should die; the second that they should be banished; and the third that they ought to be acquitted: should the two former verdicts unite their strength to

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[13] Similar motives were operative in thirteenth–century Italy. There the city republics made use of the lot and indirect election (that is, an election where the voters elect those who make the final choice). The aim "was to hinder the domination of city politics by cliques, who might prolong their control by securing the choice of members of their own faction. It was very common to combine . . . different techniques of election. This rather difficult point can only be made clearly by illustrating it. At Lucca a meeting was held in each region (*contrada*) of the city at which lots were drawn; each of the 550 men who drew slips inscribed *elector consiliarii* then had the duty of naming one man from his own *contrada* as a councillor. The podestà of Vicenza was chosen in this way (1264), after the first phase in which the Major Council determined from which city the podestà was to come. Twenty electors were drawn by lot, and of these twelve were eliminated by voting; the eight remaining electors then proposed three names, from which the final choice was made by a further vote of the Major Council"(Waley 1969 63).

the destruction of the latter? Or should not each of them separately be balanced, and the first and second be no more combined than the second and third?"(1940 2:135).[14] My intention here is just to show that the ancients were concerned with problems of voting. Indeed we might expect that such questions would arise wherever democracies or voting existed for any length of time. As far as I know investigation of voting theory and practice in ancient times from a social choice perspective is at present nonexistent. This is probably because it is widely assumed that there was no such interest until the eighteenth century. I do not know whether a thoroughly researched work would be long or short which concerned pre–eighteenth century discussions of logical difficulties with regard to voting and preferring. It might be claimed that historical precedents of this sort are not important. Well maybe so, but no one can tell what will be found until a search is made. In any case, historical knowledge puts one in a better position to evaluate the significance of new discoveries. For example, a Condorcet winner is a person who can defeat one–by–one all other candidates in a series of two–candidate contests. Condorcet probably deserves the credit for giving the first description of such a phenomenon but it is nevertheless interesting to note that an election procedure in ancient Athens may have been devised to find Condorcet winners. (See Hansen 1987 46.) However, for the moment, let us leave aside historical questions and examine, one by one, the four conditions which Arrow places on constitutions.

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[14] I found these references through reading Russell 1983 23 and Holford–Strevens 1988 216–7. Note that, from an approval voting point of view, each judge or senator could vote for what he approved of (which might include more than one possibility) and the option getting the most votes would become the decision of the court or Senate. A tie could be decided in favor of the defendant.

2

Collective Rationality

The principle of Collective Rationality means that social choices must be derivable from a set of orderings. According to Arrow an ordering presupposes three "consistency assumptions"(1967 5): (1) Connexity (or Connectedness), (2) Transitivity, and (3) Preference Implies Choice.[15] The discussion of Preference Implies Choice will be included in our examination of the Pareto Principle. Here we will consider Connexity and Transitivity.

Connexity. To understand the concept of connexity we will need to define two terms. Consider an environment consisting of a pair of alternatives between which an individual makes a choice according to his or her system of values. The alternative chosen will be called *preferred* compared to the other possible choice. If the individual chooses not to choose between the alternatives they will then be called *indifferent.* The principle of connexity requires that for each pair of alternatives, either the first is preferred to the second, or the second to the first, or they are indifferent. Arrow asserts that connexity "can hardly be denied; it simply requires that some social choice be made from any environment. Abstention from a decision cannot exist; some social state will prevail"(1963 118). The trouble with this defense is that although we might agree that "some social state will prevail" it by no means follows that that social state is either democratically chosen or preferred.

Let us see why. First, the phenomenon of "choice against preference" makes Connexity a doubtful democratic principle. Thus force or fraud can undermine democratic choice. If a gun is held to the heads of voters to induce them to vote a certain way, each vote is in accordance with the individual's value system (that is, each prefers to live), but the resulting social choice hardly indicates a preference for the winning candidate. The social choice which prevails is neither

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[15] For ease of reference I have given a name to the third assumption which is unnamed by Arrow.

democratically chosen nor preferred. Intimidation of the voter has been common practice whether in elections under totalitarian governments or under otherwise democratic governments for certain classes of voters (for example, women in New Jersey in the 1790s or blacks in Mississippi in the 1950s). Whenever a person votes for x rather than y merely because of a bribe or a vote trade or a promise to a spouse, etc., we can hardly say that the person prefers x to y. Without restrictions on various kinds of vote trading, fraud, force, or even certain promises, there is good reason not to accept Connexity.

Second, even if all those restrictions are made -- and it is certainly no easy task to articulate them in detail -- the phenomenon of what Nicholas Rescher has called "choice without preference" shows that Connexity is still an unreasonable assumption.[16] Consider an example. Suppose at a party a bowl of popcorn is down to two pieces. A person who then chooses a piece from the bowl would normally do it without a preference for the piece chosen. Again and again in ordinary life we chose without preferring. It even happens that some people, when they are indecisive in a voting booth, let the choice depend on a flip of a coin. The choice does not indicate a preference for the candidate chosen. As Rescher put it:

> . . . when a random selection among indifferent objects is made by me, I do have a reason for my particular selection, namely the fact that it was indicated to me by a random selector. But I have no *preference* or psychological motivation of other sorts to incline me to choose this item instead of its (by hypothesis indifferent) alternatives. Such absence of psychological motivation does not entail the impossibility of a logically justifiable selection. A choice can therefore be logically vindicated as having been made reasonably even though it cannot be traced to any psychological foundation. (1969 112)

Nor can Connexity be saved by claiming that choice without preference does not exist because any choice made -- even of a piece of popcorn -- must be in accordance with the chooser's value system. For we have no reason to believe that value systems operate discriminately right down to the last details of action. Even

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[16] Rescher's apt phrase led me to "choice against preference."

if we grant such minute discrimination on the individual level, this admission is not sufficient to make Connexity true for social choice. As we have seen, when ties are decided by lot in elections, the group as a whole has no social preference (that is, the group is deadlocked) but nevertheless there is a social choice for some particular alternative. The social choice is thus not derived from the set of preference orderings (as is required by Collective Rationality) but those preference orderings *and* a random selector.

Arrow's view is apparently derived from Bentham. At least he states that the "hedonistic psychology essential to Jeremy Bentham's utilitarianism has dominated modern economic analysis . . . In this doctrine, the act of choice becomes entwined with evaluation and judgement. What is chosen must have been preferred; and what is preferred must be in some sense better"(1985 140–1). However the phenomena of choice against preference (for example, in vote trading) and choice without preference (for example, flipping a coin to break a tie) show that although "some social state will prevail," that social state need not be either preferred or better. Hence Connexity is not a reasonable restriction to place on social choice.

Transitivity. Let us begin by noting that binary relations may fall into three classes: transitive, intransitive, and nontransitive. Consider some examples from arithmetic. The relation *less than* is transitive in arithmetic, since if $x < y$, and $y < z$, then $x < z$. On the other hand, *successor* is intransitive since if x is the successor of y, and y of z, then it is false that x is the successor of z. Finally, there are nontransitive relations which are neither transitive nor intransitive. For example, the relation *is not equal to* is nontransitive. It can not be transitive since $1 + 1 \neq 3$, and $3 \neq 2$, but it is false that $1 + 1 \neq 2$. Nor can it be intransitive, since $1 \neq 2$, and $2 \neq 3$ but $1 \neq 3$.

Now let $x, y, \ldots$ be a set of alternatives appropriate for a choice. Let "P" stand for *preferred*, "I" stand for *indifferent*, and "R" stand for *preferred or indifferent*. Then the assumption of Transitivity means:

For all x, y, z, if xRy and yRz then xRz.

The assumption that preference is a transitive relation may initially have some appeal, but on reflection it can be seen to be unreasonable. Preference is in fact a nontransitive relation. It can not be intransitive since sometimes, at least, if a

person prefers candidate x to y, and y to z, the person prefers x to z. Neither can preference be a transitive relation. An example of the failure of transitivity is illustrated by a sucker bet in cards. There are many versions. Consider the simplest for sake of illustration. A set of three cards, an Ace, 6 and 8 are picked from, say, a pack with red backs; a 3, 5, and 7 are picked from a blue deck; and a 2, 4 and 9 are picked from a green deck. Each set is shuffled and placed face down. The sucker draws a card from any deck and a card shark then draws from any other deck. The high card wins (an ace is considered as a 1). The sucker thinks it is a good bet because he thinks the game is transitive, whereas whatever set the sucker picks, the shark can pick another which has 5 to 4 odds of beating the sucker. That is, red beats blue, blue beats green, and green beats red, each with 5 to 4 odds. It is clear that the shark wins because he is rational and, by being rational in this case, his preferences violate Transitivity. Thus when the shark has a choice of blue or red (because the sucker has chosen green), he prefers blue to red, when the shark has a choice of red or green (because the sucker has chosen blue), he prefers red to green, and when he has a choice of blue or green (because the sucker has chosen red), the shark prefers green to blue.

If the sucker gets tired of losing in this card game, the shark could then suggest special bingo cards, unusual dice, and other games which are not transitive (Gardner 1970 and 1974). The sucker will continue to lose money as long as he thinks transitively. That is, the sucker assumes that, if given any pair in the set, there is a better choice, then there must be a best choice from the set. He thereby becomes a "money pump" for the shark. It is sometimes said having intransitive preferences makes one a "money pump" since if an individual has z and prefers y to z then the person should be willing to pay something to exchange z for y. Then having y and preferring x to y another payment would get x. Finally, if the person prefers z to x still another payment would get z and the cycle would begin all over. This is certainly true, but in the card game it would be rational for the shark to be such a pump. Thus suppose the sucker goes first and the shark automatically gets the worse choice of the two remaining packs. The best deck goes to a third nonplaying party who could sell it. It is quite rational for the shark to pay something to make an exchange with the third party and to continue to do so indefinitely so long as the sucker keeps playing, for then the shark could pay the third party from his winnings. Notice that this example is like the "voting order" version of the paradox of voting in that the party that determines the order determines the outcome. The solution is the same, namely, the sucker should insist

that a random selector determine who makes the first choice. Then the sucker ceases to be a sucker and the game becomes fair.

This example is easily adapted to a social choice situation. Imagine that the shark is replaced by a betting club where the members of the club vote for the club's social preference. If they vote unanimously to prefer red to blue, and blue to green then, by the Pareto principle, these preferences would be the social preference. From these social preferences Transitivity would then imply the irrational social preference of red over green. If they vote rationally, however, we have a case of the paradox of voting which does not necessarily involve the fallacy of equivocation, that is, in this case a unanimous majority prefers red to blue, blue to green, and green to red. It is obvious that it is the same majority each time and thus there is no equivocation on "majority." Again, however, there is no logical or philosophical paradox.

We can also adapt the same example to politics. Suppose in the minds of voters three candidates for mayor A, B, C are ranked (on a scale of 1 to 9) according to their ability in three categories: administration, leadership, and intelligence. To simplify the illustration assume their rankings are by chance unanimous and unchanging over the years. Let the ranking be as follows:

	Administration	Leadership	Intelligence
Candidate A	2	7	6
Candidate B	9	5	1
Candidate C	4	3	8

Let us imagine that there are yearly elections over a long period of time where only two candidates run (chosen at random). If the voters want to pick the candidate who is superior to the others in at least two categories, then it is rational for them to prefer A to B, B to C, and C to A. Notice too that we need not suppose that individual voters tell anyone else about their numerical rankings -- all they need to reveal by their behavior is their preference when voting. In any case what this example shows is that there is no best candidate overall even though there is a better candidate in any pair. In other words, it is quite rational for society to have a definite preference with regard to any pair, but to have no definite preference with respect to all the candidates.

Violations of Transitivity also occur where the differences among options with regard to one quality are sometimes insignificant, and the choice must then

be made on the basis of another quality.[17] Consider admission to a technical school. The most important quality, we will assume, is the score of the Scholastic Aptitude Test in mathematics. Differences between scores of ten or fewer points are, however, not considered significant. In such cases the decision is made on the verbal aptitude. Experience has shown here that any score over 400 on verbal aptitude is satisfactory and, again, that differences between scores of ten or fewer points are not significant. Let us imagine that three students must be picked from a total of seven and the following situation holds:

Student	Mathematics	Verbal
A	600	500
B	602	488
C	604	476
D	606	464
E	608	452
F	610	440
G	612	428

The admissions officer of the school is then being rational in preferring A to B, B to C, C to D, D to E, E to F, F to G and G to A. Such a situation reveals social indecision and the use of a random device may be appropriate. Another reasonable method of choice would introduce new criteria. For example, the admissions officer might admit students A, D, and G to see if their subsequent performance would suggest refining the present criteria. Or the officer might use appeal to hitherto irrelevant factors such as home state, minority or sexual status, athletic ability, and so forth.

Arrow's condition of Transitivity is contrary to the empirical temper. For example, in sports it is notorious that transitivity of preferences is violated. A sports betting club whose social preferences were always transitive would sometimes be acting irrationally. Or consider a chess club that wants to buy the best chess–playing machine. There are three choices: *A*, *B*, and *C*. Investigation shows that *A* beats *B* and *B* beats *C*. Hence the club votes unanimously to prefer *A* to *B* and *B* to *C*. Is it reasonable for the club then, on the basis of *a priori*

[17] This point, and the inspiration for the example which follows, comes from Tversky 1969. However, the use I make of it is not the same.

democratic voting principles alone, socially to prefer *A* to *C*? Might not *C* beat *A* and, as a result, the club socially prefer *C* to *A*? Thus there is no best chess machine in the triplet *A*, *B*, *C*, although there is a better machine in any pair from the triplet. Again, the choice can be made on the basis of a hitherto irrelevant factor such as price. It is surely unreasonable for democrats to adopt a principle of social choice which prevents them from being rational in some circumstances, and then to make that principle a necessary condition for another principle called Collective Rationality.[18]

One need only produce one counterexample to show that preference is not transitive. If someone claimed that "love" was transitive, it would be sufficient to produce one counterexample to demonstrate the claim's falsehood. To be effective the counterexample would have to involve an unmistakably clear case of love. Arrow's taking the principle of Transitivity as part of the standard of social choice runs the risk that some actual social choices will be based on the same kind of error which makes one a sucker in gambling. He states that the "idea of transitivity clearly corresponds to some strong feeling of the meaning of consistency in our choice" and any person or community which violates it is not "behaving rationally"(1967 5; 1963 3,19). We can certainly agree that some people feel strongly that preference must be transitive. To me this is like a strong feeling that the set of positive integers is larger than the set of even numbers. One may have such a strong feeling but an analysis of what one means by "larger" (*à la* Cantor) shows the feeling to be misleading. Similarly an analysis of preference shows that the feeling to which Arrow refers is misleading and that any person or community which was uniformly transitive in its preferences would be irrational, for preference, like love, is nontransitive.

Even apart from the problems of Connexity and Transitivity, there are other difficulties with the principle of Collective Rationality. Consider the following

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[18] It should be noted that judges sometimes have to face violations of the principle of Transitivity in the law courts. In law *prefer* means to give priority to one creditor over another. It sometimes happens, with regard to an inadequate fund, that A has priority over B, B has priority over C, and C has priority over A. The judge, who must make the social decision, finds that the law requires him to prefer A to B, B to C and C to A. Such a situation might mean that the legislature was careless or indecisive or wished to give the judge discretion in such cases. At any rate, this predicament was recognized at least as far back as 1685. It has produced an extensive literature which so far has unfortunately been ignored by social choice theorists. There is a bibliography on this topic ("circularity of liens") by Peter Suber and additional references can be found in the articles themselves. See Bartlett and Suber 1987 348–9.

quotation from Charles Sanders Peirce:

> When we busy ourselves to find the answer to a question, we are going upon the hope that there is an answer, which can be called *the* answer, that is, the final answer. It may be there is none. If any profound and learned member of the German Shakespearian Society were to start the inquiry how long since Polonius had had his hair cut at the time of his death, perhaps the only reply that could be made would be that Polonius was nothing but a creature of Shakespeare's brain, and that Shakespeare never thought of the point raised. Now it is certainly conceivable that this world which we call the real world is not perfectly real but that there are things similarly indeterminate. We cannot be sure that it is not so. (1933 4:41)

Peirce's great insight found confirmation many years later in the works of Heisenberg in physics, and of Gödel in logic. Applied to Peirce's example, the spirit of Arrow's Collective Rationality condition comes to the following: All answers to haircut questions about Hamlet must be *derivable* from the text of *Hamlet*. Anyone asserting this principle would be confused, for he would be making an *a priori* claim about something which can only be determined after an analysis of the Shakespearean text. And some questions will have no answer on the basis of the text. The principle of Collective Rationality comes to the following claim: all democratic social choice questions must be *derivable* from the preference orderings of the individuals making up the democratic group. This claim can only be substantiated by an analysis of the practice of democracies and the meaning of the term "democracy." Even without getting into the details of that analysis we can see difficulties. Consider an analogy. Suppose someone, using generally accepted methods of mathematics, proves Fermat's last theorem.[19] It would not follow that it is derivable from the axioms of arithmetic. It might be undecidable using only those axioms, but be decidable using methods derived from analysis. Similarly, if some community, using generally accepted democratic methods in an exemplary fashion, takes a given social action it might not be derivable -- even in principle -- from the preference orderings of the individuals

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[19] "Fermat's last theorem" refers to the still unproved conjecture (made over 350 years ago) that $a^n + b^n = c^n$ is always false if a, b, c, n are positive integers and n is greater than 2.

making up the group. As we have seen, this is the case when random selectors are used to resolve tie votes.

In sum, the principle of Collective Rationality presupposes an ordering which requires the three assumptions of Connexity, Transitivity, and Preference Implies Choice. Connexity must be qualified to rule out antidemocratic choice against preference (force, fraud, etc.) and is incompatible with democratic choice against preference (vote trading) and democratic choice without preference (random selectors). Transitivity prevents democratic social groups from being rational in some situations. And Peirce's haircut example shows that Collective Rationality asserts more than we have a right to claim. These considerations allow us to conclude that Collective Rationality is an unreasonable principle. However, the argument against Preference Implies Choice and thereby the completion of the argument against Collective Rationality depends on a further analysis which will be taken up in the next chapter.

3

Pareto Principle

Imagine a professor is teaching a course in logic and the topic is the fallacy of composition. The professor is using Irving Copi's elementary textbook and quotes the following example: ". . . since every part of a certain machine is light in weight, the machine 'as a whole' is light in weight"(1986 117). The teacher then asks: "Can any one illustrate the fallacy with respect to the property of mortality?" A student answers: "If every presently living American ceases to exist before 2776, then American society will cease to exist before 2776." "Right," the instructor answers and immediately asks: "Can anyone give an example using the concept of a preference ordering?" A student answers by quoting the Pareto Principle: "If alternative x is preferred to alternative y by every single individual according to his ordering, then the social ordering also ranks x above y." Would the professor be justified in saying "Right, again!"? Perhaps some such reasoning may have led Arrow to the principle. For Arrow states that the "hedonistic psychology associated with utilitarian philosophy" is meant to be embodied in his system by having the social ordering relation depend only on individual ordering relations. "The hedonist psychology . . . was . . . used to imply that each individual's good was identical with his desires. Hence the social good was in some sense to be a composite of the desires of individuals"(1950 335). And Copi (in an exercise) gives the following quotation from John Stuart Mill as an example of the fallacy of composition: ". . . each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons"(Copi 1986 123; cf. Mill 1948 33). Arrow seems to be making a statement similar to Mill's, that is, he seems to be claiming, *via* the Pareto Principle, that in a democracy if all the individuals have a certain property (a particular preference ordering) then

the democratic whole made up of those individuals must have that property.[20] Arrow might plausibly be defended by saying that the Pareto Principle is prescriptive and therefore it merely states what *ought* to be the case in a democracy. Then, however, the Principle is open to the charge of committing the fallacy of *consensus gentium.*

Arrow's motive is presumably to deny that there can be a standard of value which we can discover independent of values which individual human beings can be shown to hold. But that does not mean that we can ignore the way in which a person or society comes to hold particular preferences. And it is on this point that we can see the weakness of Preference Implies Choice. Arrow characterizes this assumption as follows: ". . . it is assumed that the choice from any environment is determined by the ordering in the sense that if there is an alternative which is preferred to every other alternative in the environment, then it is the chosen element"(1967 5). Yet it is unreasonable to assume that any possible social preference would be an appropriate *democratic* social choice. For instance, suppose a vote is taken where the entire electorate was clearly misinformed about some important, relevant fact. Must their unanimous vote become the social choice? Consider a tax–exempt organization that votes unanimously to raise money through a bingo game. However, the president of the organization later finds out that this is illegal. Is the president obligated by democratic principles to go ahead with the bingo game?

Nor can any social preference ordering become the democratic social choice when that ordering contradicts the ideals of democracy. Consider three examples. Suppose that everyone in a particular country prefers that any foreigner crossing the border should be executed. Several hundred children in a neighboring country are forced across the border by a civil war there. Yet to execute the children as everyone prefers is obviously contrary to democratic principles such as the unalienable right to life asserted in the Declaration of Independence. Again, imagine that in a particular community some persons decide to contract themselves into slavery. As long as everyone agrees the Pareto Principle would require that these contracts be valid, contrary to the Constitution and the claims of such democratic philosophers as John Locke (1988 284) and John Stuart Mill (1978

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[20] As we have seen the claim that preference ought to be transitive may also rest on a fallacy–of–composition type inference: If, given any pair from a set of objects, society has a definite preference, then society must also have a definite preference with respect to the entire set.

101–2). As a final example consider a society -- call it country A -- where a man called Malcolm commits a particularly gruesome crime and is immediately caught. Everyone prefers that he be executed by sundown -- including Malcolm who is a fanatic and who intends to be a martyr. Abiding by the Pareto Principle and Preference Implies Choice, the social decision leads to immediate execution. Let us imagine country B has an identical situation except that there the social choice process -- contrary to universal preference -- leads to Malcolm receiving a meticulously fair trial which lasts several months. Accepting the Constitution and the *Federalist Papers* would require that we prefer country B (which gives a meticulously fair trial to Malcolm) to country A (which lynches Malcolm by sundown), whereas the Pareto Principle leads to the contrary judgment. Arrow once expressed the Pareto Principle as follows: "the social choice process shall never yield an outcome if there is another feasible alternative which everyone prefers according to his preference ordering"(1974a 270). Applied to our example this becomes: the social choice process shall never yield a fair trial for Malcolm if immediate execution is a feasible alternative which everyone prefers according to his preference ordering. The Pareto Principle would thus authorize the lynching of Malcolm, which is just the kind of community choice that any democracy ought to prevent by effective institutional safeguards.

Perhaps an analogy to the science of medicine would be useful here. The Nazi regime used doctors to devise tortures and execute large numbers of innocent victims including children. Should we claim that those doctors were practicing medicine provided they were using what they learned in medical school? Is it not, rather, a perversion of medicine? Democracy likewise presupposes a set of moral values. Yet the morally neutral Pareto Principle would *require* the violation of these values in any society where everyone agreed to the violation.[21] Should we claim that such a society is acting democratically? Or is this not a perversion of democracy? If the moral values are safeguarded (by bills of rights, checks and balances, etc.), then the Pareto Principle may be a reasonable democratic assumption. Yet it is hard to see how one can support the Declaration of Independence, the Nuremberg trials, or the work of Amnesty International and also support the unqualified form of the Pareto Principle which Arrow assumes. Again, when the federal government imposes low–cost housing on a community against

[21] See van den Haag 1967 for further discussion of this point.

its will, or when the Allies after World War II imposed democratic institutions on Germany or Japan against the consent of the Germans and Japanese, a democrat is hardly obliged to condemn these acts. John Adams asserted the utilitarian conception of government when he said: ". . . the end of government is the greatest happiness of the greatest number" but he added a qualification, namely, that "at the same time the stipulated rights of all" must be saved (1794 1:29). The qualification is imperative, for democratic communities, no less than individuals, know that in the future they may have weak moments and ought to prepare for them so that, when a weak moment arrives, they are less likely to violate their own standards of morality and rationality. Thus democrats have created a variety of devices, practices, and institutions whose purpose is to protect the people against the government and the people against themselves (self–paternalism). These include the legislative, executive and judicial branches of government, bicameral legislatures, criminal trial procedures, judges appointed for life, multiple schemes of voting, minimum standards for the education of children, and so forth. In a democracy it is important to know how an individual or a community forms the preferences they have.[22] As James Madison put it:

> As the cool and deliberate sense of the community ought in all governments, and actually will in all free governments ultimately prevail over the views of its rulers; so there are particular moments in public affairs, when the people stimulated by some irregular passion, or some illicit advantage, or misled by the artful misrepresentations of interested men, may call for measures which they themselves will afterwards be the most ready to lament and condemn. (1962 10:546)

A Madisonian democracy -- what he called a republic -- is structured to insure the triumph of critical reason over political desire.[23] In such a republic the social good can never merely be a "composite of the desires of individuals"(Arrow 1950 335). Those desires must first pass the tests of self–paternalistic democracy in

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[22] Even in mathematics the significance of a result may depend on how it is reached. Thus identical theorems, one which was reached by using the law of the excluded middle applied to infinite sets and one which was reached by purely finitary means, may not have the same import.

[23] "But it is the reason of the public alone that ought to controul and regulate the government. The passions ought to be controuled and regulated by government"(Madison 1962 10:463).

order to count toward defining the social good. Of course, what actually constitutes legitimate self–paternalism in a democracy –– Madison was talking about the Senate –– is open to debate. But the very concept of democratic self–paternalism is contrary to both Preference Implies Choice and the Pareto Principle. Some social preferences should not become the *democratic* social choice, and this holds even if those social preferences are supported by every individual in the democracy. Before the democratic ideals represented by fundamental political rights, the mighty Pareto Principle must always curtsy and, sometimes, it must be dismissed.

In sum, the Pareto Principle, is either an example of the fallacy of composition or *consensus gentium.* In conjunction with Preference Implies Choice, the Pareto Principle can justify acts which are contrary to the ethical basis of democracy. Both principles are inconsistent with the self–paternalism which democratic governments need to have in order to protect the fundamental rights of democracy. Unless they are substantially qualified, therefore, these principles are not reasonable ones for a democracy.[24]

[24] For further arguments against the Pareto Principle, see Sen 1982 (Part IV).

4

Independence of Irrelevant Alternatives

Arrow gives the following illustration of the Independence of Irrelevant Alternatives: "Suppose, for example, that a community has to choose between the construction of a stadium and that of a museum; it is capable of financing either one, but not both. Suppose further that a university is in any case beyond the means of this community. It seems evident to me that the choice between the museum and the stadium should be independent of the preferences of the members of the community between a museum and a university. The essential argument in favor of this principle is its direct appeal to intuition"(1983 1:51). I agree with Arrow that the principle has intuitive appeal but one must be wary about depending on one's intuition. That *the whole is greater than the part* has great intuitive appeal -- in fact, it is Euclid's Fifth Common Notion -- but a consideration of infinite sets shows it to be false. We can note again that the number of elements in the set of positive integers (the whole) is not greater than the number of elements in the set of even numbers (the part). One can also find democratic counterexamples to the Independence of Irrelevant Alternatives.

Let us suppose a betting club of five people (who call themselves BET) goes to a booth at a carnival and decides to bet on some wheels of fortune. We will assume that the social preferences of BET are the result of unanimous agreement of all the members. In the booth there are three wheels, each with a hundred equal divisions.[25] Each division on each wheel has a certain value. On wheel A each division is worth 3; on wheel B, 56 are worth 2, 22 are worth 4, and 22 are worth 6; on wheel C, 51 are worth 1 and 49 are worth 5. BET chooses a wheel and then the operator of the booth chooses one of the two remaining wheels. Then they are spun and the highest number wins. In trying to decide what wheel to pick BET begins by setting up the following table:

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[25] The example which follows is a adaptation of one used by Blyth 1972 371 in another connection. Also see Gardner 1976 which influenced me.

Wheels			Equations	Ranking		
A	B	C	A B C	1st	2nd	3rd
3	2	1	1 x .56 x .51 = .2856	A	B	C
3	2	5	1 x .56 x .49 = .2744	C	A	B
3	4	1	1 x .22 x .51 = .1122	B	A	C
3	4	5	1 x .22 x .49 = .1078	C	B	A
3	6	1	1 x .22 x .51 = .1122	B	A	C
3	6	5	1 x .22 x .49 = .1078	B	C	A

From this table BET correctly concludes that A wins 28.56% of the time; B wins 33.22% of the time [.1122 + .1122 + .1078 = .3322]; C wins 38.22% of the time [.2744 + .1078 = .3822]. Hence BET ranks the social choice of the club C, B, A and thus asks to spin wheel C.

The operator of the booth announces that wheel B is broken and thus unavailable. Because BET assumes the Independence of Irrelevant Alternatives, BET believes its social preference for C over A does not depend on its view of B. Hence BET makes no new calculation but merely deletes B from the preference list to get a new preference list C, A. BET thus again asks to spin wheel C. BET's assumption is wrong. If it is a choice between wheel A and wheel C, then A is the better choice since it beats C 51% of the time. The judgment of whether A or C is better depends on whether the "irrelevant" B is taken into account.

Now consider another illustration by Arrow of the Independence of Irrelevant Alternatives. Assume that an election with more than two candidates is held. Each voter fills in his or her complete list of preferences. A candidate then dies. The social choice is made on the basis of the same set of preferences where the name of the dead candidate has been deleted from each list. (See 1950 337.) However, this procedure did not work for BET and it will not work here. For instance, in an election of three candidates for president of an organization, suppose the situation is as follows: First, A and C have each served on a number of occasions as president. The membership believes that A is the better president, but C works poorly with others who are in authority but not with others when he is president. When C is not president he is such an obstructionist that the meetings are miserable. Hence the membership all believe that C will work out better as President since A is cooperative. Second, B joins the group and remains as a member for some years. B has such a strong personality that when B is present

C is actually cooperative with A. After B joins the group, the membership prefers A to C for President. Third, B finally decides to run. The membership -- fearing B's strong personality in a position of power -- prefers A to either B or C. However, just before the election B dies. In this case it would be unreasonable to delete B from the preference lists and let A be the social choice. If B dies, all the members should report that they prefer C to A. To deny Irrelevant Alternatives, Arrow tells us, would make the election's outcome "dependent on the obviously accidental circumstance of whether a candidate died before or after the date of polling"(1950 337). One can only say, the election's outcome can and should depend on such accidental circumstances. Of course, B does not actually have to die or be unavailable. It is sufficient for him to announce that he will cease coming to meetings if he is not elected. The Independence of Irrelevant Alternatives forces voters to assume *a priori* that there are no relations among the alternatives (say, B vis-a-vis A or C) which they might consider relevant in their choice of A or C. But this is for the voters to decide *a posteriori*.

Finally, "irrelevant" is not exactly defined. If we imagine the cost of the university, in Arrow's example, decreasing by a penny is it still irrelevant? Surely, but as this process is continued there comes a point where the question of whether it is relevant or not is unclear and thus whether the Independence of Irrelevant Alternatives applies is likewise unclear. Nor is the case any better with respect to "feasible." In a federal system it is quite useful to consider alternatives which are not feasible. If each person in the community expressed a desire for a university (though none could afford it) the state government (which could afford it) might then authorize the project. As a further example consider the Fifth Amendment according to which "no person shall. . . be deprived of life, liberty, or property, without due process of law. . ." It might plausibly be argued that this is not feasible since it is an impossible to prevent all murders, thefts, etc.. If so, when Congress voted on the Bill of Rights did it violate the requirement of feasibility? Might not the construction of a society in which the Fifth Amendment holds be as far beyond the means of Congress as the construction of a university is beyond the means of Arrow's mythical community? Should it then have been rejected in favor of alternative amendments which were actually achievable at the end of the eighteenth century? Again, suppose a liberal and a conservative are running for President but the election is thrown into the House of Representatives because an ultraliberal write-in candidate garners 24% of the vote. We will assume that the write-in candidate is only thirty-four and thus ineligible to serve. In such

circumstances is it credible that the choice which the House makes between the liberal and the conservative ought not to be influenced by their knowledge of the extent of the public's support for the ultraliberal?

Notice that the BET example does not violate the requirement that preferences may have only ordinal significance and the intensities of preferences may not be compared by the social choice theorist. (For example, see Arrow 1967a 231.) BET reports on the *ordering* of preference, not the *strength* of the preference. The requirement that preferences have only ordinal significance would be absurd if it would prevent voters from noticing that one bond issue is twice as expensive as another, or would prevent members of Congress from comparing the size of budgets when they vote. Cardinal comparisons are in any case a central feature of representative democracy even if no cardinal assumptions are made about the voters. For example, suppose there are a hundred districts each with a 1000 voters who consider two positions on some issue. In each of 49 districts the voter tally is exactly the same: 999 for A and 1 for B. In each of the remaining 51 districts the tally is 501 for B and 449 for A. In such a situation it is reasonable to give a cardinal interpretation to the strength of the social preference of A over B (or B over A). The stage is set for representatives of the districts to engage in vote trading with respect to this issue and some other proposal. Hence a cardinal interpretation emerges in representative government even if each and every ballot has only ordinal significance. Indeed if vote trading is allowed for individuals, then the intensity of preference could be the object of empirical study where, for example, one would make the assumption that a voter who trades her right to vote on twelve issues for the right to vote twelve times on some other issue cares more about the latter issue than she cared about the others. One could then compare her pattern of vote trading with that of others.

The Principle of the Independence of Irrelevant Alternatives is not actually used in Arrow's proof which makes no reference to feasible alternatives or changes in the set of alternatives. Instead another, related principle is used in which even feasible alternatives are irrelevant. It is sometimes called Pairwise Determination. Arrow, however, uses the same name for both principles. Pairwise Determination refers to changes in preference orderings. Arrow describes it as follows: ". . . if we consider two sets of individual orderings such that, for each individual, his ordering of those particular alternatives under consideration is the same each time, then we require that the choice made by society be the same if individual values are given by the first set of orderings as if they are given by the second"(1950

337).[26] But if the orderings create a symmetrical deadlock, then the random device used to break the deadlock might lead the social choice to go one way for the first set of orderings and another way for the second set. Is it reasonable to build into the constitution of democracy a prohibition against any use of random procedures when they have proven themselves -- beginning thousands of years ago -- to be extremely useful in promoting democratic ideals? For example, the decision to use a random device for the draft represents a commitment to democratic equality and it seems unreasonable, on social choice principles alone, to rule out the practice.

In sum, the trouble with Independence of Irrelevant Alternatives in its feasibility version is that taking into account another alternative, even if that alternative is not chosen, may change the voting game for rational voters. The principle in its pairwise determination version is inconsistent with the common democratic practice of using a random procedure. Finally, it is rational for representatives in legislatures to violate it. It is not, therefore, something reasonable for a democrat to assume.

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[26] Arrow himself points out the difference in the two versions of the Principle. See Feiwel 1987 195. However, since he continues to describe the first version more than twenty-five years after the first publication of his proof I thought it best to criticize them both. See Arrow 1977 614.

5

Nondictatorship

Let us imagine that in the sixteenth century there was a shipwreck and several hundred people were forced to live on a previously uninhabited island for several years. They are all adults and unanimously agree that to preserve equality social decisions will be made by a dictator who is chosen randomly the first of each month. No person can be a dictator more than once. After two years and twenty–four dictators they are rescued by a passing ship at which time all agreed that their political system worked well and that fundamental human rights had in fact been protected. The islanders' political system violates both Collective Rationality and Nondictatorship. Nevertheless, since it protects fundamental human rights and uses the ancient democratic practice of choosing leaders at random to preserve equality, it certainly deserves to be called a democracy.[27] Nor would the case be any different if they were rescued in one month having had just one dictator. The objection against a dictator is not that his existence is intrinsically incompatible with democracy. Rather, the objection is with the ordinary ways dictators are chosen or disposed, and that in many circumstances dictators have not protected fundamental human rights. The objection against the use of lot is that quite often leaders are chosen who are unwilling or incompetent to serve. Where these circumstances do not apply –– say, in some clubs or unions or businesses –

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[27] For instance, the following is from Aristotle's *Politics*: "This is the second characteristic of democracy [that is, that a man should live as he likes], whence has arisen the claim of men to be ruled by none, if possible or, if this is impossible, to rule and be ruled in turns; and so it contributes to the freedom based on equality." (Aristotle, 2:2091 [1317b 11–16]) And here is a passage from the *Constitution of Athens*: "There is a single President of the Prytanes, elected by lot, who presides for a night and an day; he may not hold the office for more than that time, nor may the same individual hold it twice"(2:2369 [Section 44]). It is sometimes claimed that the Athenian conception of democracy is fundamentally different from a modern one in that political ideals (such as liberty or equality) played little or no part in the former. This view is convincingly controverted in Mogens Herman Hansen's *Was Athens a Democracy?* which emphasizes the similarities between the two conceptions.

– there is no democratic objection to the procedure of regularly choosing dictators by lot. Indeed American democracy began with a Declaration which asserts it is the people's right to choose any form of government which best protects their fundamental rights. It is very important not to confuse properties often associated with democracy, such as voting or free markets, with its essence, which is the protection of the rights of the people and the achievement of certain moral ideals. Hence I would disagree with Arrow when he claims that essentially only voting and the market are available for making social choices in a "capitalist democracy"(1963 1). In fact, in American democracy many, many social decisions are not made either by voting or the market. Instead they are made by bureaucrats, military officials, judges and random selectors.

Voting is certainly one of the important means used in a democracy to insure the protection of rights. But it must sometimes be dispensed with to protect minority rights. For example, if members of a community voted that a certain citizen should not be able to speak in public, or must worship at a certain church, then the courts should hold the outcome of these votes to be void. Or consider a simple, even trivial, example. Suppose at a summer camp fifteen boys must choose either cake or ice cream for dessert each evening. Circumstances are such that they must all have cake, or all have ice cream. Suppose each night they vote and it is always eight to seven in favor of ice cream. We assume it is the same eight boys who want ice cream each night. If they vote each evening the minority will not get any cake all summer -- their right to equal consideration is not being protected. But there are democratic alternatives which do not involve voting. For example, if there are sixty suppers, one by one each boy could be made "dictator" for a night and choose the dessert. Each would thus get four chances to choose over the summer. Alternatively a random device could be used to pick a boy each evening who would then choose the dessert. To identify voting and democratic decision procedure is wrong historically (ancient Athens), and it is wrong logically (sometimes random procedures are needed to break symmetric deadlocks), and it is wrong with regard to contemporary democratic practice (for example, appointed judges who have tenure for life). Democracy is the attempt to make the practice of government consistent with the view that each person has an intrinsic dignity and is born with equal moral worth to any other. Voting is often a good means to attain that consistency, but sometimes it is dispensable, and sometimes it is incompatible, with that aim.

Arrow states that "the desire for a dictatorship or for a particular dictator

may be overwhelming under certain conditions. In such a case our social welfare problem may be regarded as solved. . ."(1963 90). This statement is in general correct but needs some qualification. First, the social welfare problem might not be solved since the dictator might be indecisive, perhaps because he has a tripartite psyche such as Plato or Freud proposed, and is caught up in an intrapsychic "paradox of voting." Second, if the kind of dictatorship which is desired is incompatible with the moral ideals of democracy then the problem for the democratic social choice theorist is to design schemes to prevent the community from acting on those desires, however overwhelming they might be. For example, Article IV, Section 4 of the Constitution states that the "United States shall guarantee to every state in this union a republican form of government . . . " Hence if the people of a certain state have an overwhelming desire for a dictatorship which is incompatible with republican rights, then the Federal government is legally obliged to prevent the creation of that dictatorship.

In sum, just as democratic decisions should not be limited to voting and the market mechanism, neither should democracy necessarily preclude the use of a dictator. What is important is why and how the dictator is chosen, and why and how the dictator is disposed. The shipwreck example shows that democracy is compatible with a series of dictators. Hence, the principle of Nondictatorship is mistaken because it presupposes that a property of many democratic decisions (the absence of a dictator) is an essential property of all democratic decisions.

6

The Invalidity of Arrow's Argument

We can conclude that Arrow's conditions are unreasonable. They certainly cannot be considered true of Athenian or American democracy. But suppose for the sake of the argument we accept all of them. Are we then logically committed to Arrow's theorem? The answer is still "no" since the argument from those premisses contains logical errors. To expose them let us begin by considering the following definitions:

> $x\bar{D}y$ means I is a *dictator* with respect to x and y, that is, if I prefers x to y then, irrespective of the preferences of all others, society prefers x to y.
>
> xDy means I is *decisive* with respect to x and y, that is, if I prefers x to y and all others prefer y to x, then society prefers x to y.

The proof (1963 98–100) is divided into two parts. The first section of Part I attempts to establish the following statement for arbitrary alternatives x, y, z: If I is decisive with respect to x and y, then I is a dictator with respect to x, y, and z. The second section of Part I purports to extend this result to any number of alternatives and concludes that, for any individual I, and any x and y, xDy is false.[28] Part II then endeavors to show, via the paradox of voting, that there is no social choice mechanism satisfying the four conditions.

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[28] Note that this result should make us skeptical that Arrow's assumptions are reasonable for a democrat to accept even before a paradox–of–voting type of argument is made in Part II of the proof. For democracies give authority to various officials (fire chiefs, presidents, IRS examiners, et al.) to make decisions regardless of the opinions of everyone else. Then various structures prevent that individual from assuming more authority than is granted to anyone by occupying that position. These structures are thus contrary to Arrow's assumptions and it suggests that something is wrong with those assumptions independent of any appeal to the paradox of voting.

Now let us examine what is wrong with the first section of Part I. The initial segment of that section aims at establishing that if xDy then $x\bar{D}z$. The argument is as follows: Assume xDy is true for some x and y, and there is only one other alternative z. Let I's preference ordering be x, y, z. Assume that each individual other than I prefers y to x and y to z, but may prefer x to z, or z to x, or be indifferent between x and z. Then x is socially preferred to y since xDy and I prefers x to y while everyone else prefers y to x. By the Pareto Principle y is socially preferred to z since everyone prefers y to z. Then by Transitivity x is socially preferred to z. Hence $x\bar{D}z$ holds since x is socially preferred to z regardless of the preferences of anyone between x and z other than I. So if xDy then $x\bar{D}z$.

The error in this argument comes in the assumption of an arbitrary set of alternatives x, y and z. Note that, given all the assumptions, x is the social choice. But suppose we introduce a self–referential argument by letting x stand for the repeal of Transitivity or the Independence of Irrelevant Alternatives. Then the proof is invalid since the argument by which the conclusion was established depends on those principles applying to the social choice mechanism.[29] Arrow states that his conditions on constitutions are "value judgments and could be called into question"(1950 339). Yet he does not discuss the consequences if the voters themselves unanimously reject one or more of them in the social choice mechanism itself. Nor does he discuss the implications if x stood for the requirement that a random selector for social choice must be used whenever there is a symmetrical deadlock in social preference. Such a possibility would raise difficulties for Part II of Arrow's argument since it depends on the paradox of voting.

Arrow's argument might be defended by saying that principles such as the Independence of Irrelevant Alternatives are not subject to change and thus not appropriate for the content of a motion. But would not this defense employ the sort of Platonism which Arrow rejects? Or can human beings create unchangeable

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[29] Self–referential arguments in law have been known since ancient times. See Gellius 1927 1:404–8, where the Protagoras–Euathlus debate is described. The paradox of self–amendment is especially worthy of study in connection with Arrow's proof. An excellent account of this topic can be found in Suber 1990. See also the exposition of part of Suber's work in Hofstadter 1982, and a related article "Logical Rudeness" in Bartlett and Suber 1987 41–67. If democracy contains fundamental logical problems the paradox of self–amendment may be one of them. My self–referential objection to Arrow's argument was inspired by the analogy to Gödel's argument in mathematical logic and by Arrow's claim that he is interested in "the entire system of values, including values about values"(1950 334).

social objects? If so, how? Further, is such an attitude democratic? Must a professional organization, say, in order to be democratic, operate according to the Independence of Irrelevant Alternatives where every member objects to it? Article V of the American Constitution states that one of its provisions cannot be amended: "no state, without its consent, shall be deprived of its equal suffrage in the Senate." Although the prohibition against amendment without a doubt makes it more difficult to change equal suffrage in the Senate it is nevertheless a false claim since the Constitution can be amended by deleting the prohibition and then making the change. Constitutions are mutable.[30] If better principles of democratic social choice than have hitherto been known are discovered, can not a democracy adopt them? Democracies must be able to revise their own basis, for example, so as better to secure the moral ideals and political rights for which democratic institutions were devised in the first place. Could the people of a democracy be called autonomous if they did not have this power? Hence the principle of the Independence of Irrelevant Alternatives -- even if unobjectionable according to present thought -- ought not to occupy as secure a place in the structure of political democracy as that enjoyed by moral ideals and fundamental political rights. Democratic social choice is best represented not as a fixed formal system but rather as a dynamical one in which both the rules and the players may change (human beings, after all, are mortal), but the goal of achieving democratic ideals remains.

Even if we assume that democracies have no right to change Arrow's four conditions there are still substantial difficulties in finding language which rules out all implicit or explicit self-referential arguments which might upset Arrow's proof. The arguments of Gödel, Rosser and others have shown that self-reference can be quite subtle indeed. Consequently, even under this assumption, we ought to be skeptical that Arrow's proof is valid until we see that language.

Arrow's proof is undermined by a self-referential argument but that is not the only logical error in the proof. There is another: the application of the replacement property of equality in an intensional context. The *replacement property of equality* means that if x is identical to y, then in any context in which "x" occurs, "y" may replace it without altering the truth value of the context. For example, since $2 + 2 = 4$ it follows that neither the truth of $2 + 2 > 3$, nor the

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[30] " . . . one of the most important and well known facts of politics is that institutions of social decision making are mutable"(Friedland and Cimbala 1973 57).

falsehood of $2 + 2 < 1$, is upset by the replacement of "2 + 2" by "4" to get, respectively, $4 > 3$ and $4 < 1$. Contexts in which the replacement property of equality hold are called *extensional*; otherwise, they are called *intensional*. Since the preference predicate is intensional, the following argument is invalid:

Harry prefers c to b.
$a = c$.
Therefore, Harry prefers a to b.

As an example let b and c be two different brand–name aspirin and a be a generic aspirin. We can assume that both b and c are acceptable to Harry (although he prefers c) but he thinks generic aspirin is inferior and refuses to use it. We will also assume the second premiss is true but its truth is unknown to Harry. Then the premisses are true and the conclusion is false.[31]

Now consider a final example. The President (who will be individual I) has sole authority (we will suppose) to appoint the Chief of Naval Intelligence. Assume that every adult American citizen (excepting the President) prefers Baker (b) to Able (a) for the job. The President nevertheless publicly names Able to succeed the person who will retire from the job in six months. Hence aDb is true since the President has sole authority in this matter. We will need two abbreviations:

x = the principal spy in America from the Soviet Union
y = the most talented American intelligence officer

We will assume $a = x$ is true but this fact is known to no American except possibly the President. $b = y$ is also true but it is known only to a few people in the intelligence community. Finally, consider the following argument:

aDb	Premiss
$a = x$	Premiss

[31] This is an adaptation of an example Arrow uses in another context. See Arrow 1963 87. Elsewhere (page 46) in the course of a proof about majority decision Arrow assumes xRy and yRz. He states that "the conclusion xRz is trivial if $x = y$ or $y = z$". Far from being trivial this conclusion is invalidly drawn. For a discussion of this kind of logical error, see Kleene 1967 163–4.

$b = y$	Premiss
Therefore, xDy	Replacement property of equality

By our assumptions each of the three premisses is true. However, the conclusion is false. If the President publicly stated that he preferred the principal spy in America from the Soviet Union for the Chief of Naval Intelligence to the most talented American intelligence officer he would either be impeached for treason under Article II, Section 4 of the Constitution or declared incompetent under the Twenty–Fifth Amendment. The principal spy in America from the Soviet Union would certainly not be installed as Chief of Naval Intelligence six months hence. That is, the President is decisive with respect to a and b, but he is not decisive with respect to x and y, even though $a = x$ and $b = y$.

An argument of this form appears in the second section of Part I of Arrow's proof. If we grant the argument in the first section of Part I then the following statement (which Arrow calls "(9)") is established:

If xDy, then $u\bar{D}v$ holds for every ordered pair u, v from the three alternatives x, y, and z.

The argument proceeds,: "Suppose aDb holds, and let x and y be any pair of alternatives. If x and y are the same as a and b, either in the same or in the reverse order, we add a third alternative c to a and b; then we can apply (9) to the triple a, b, c and deduce $x\bar{D}y$ by letting $u = x$, $v = y$"(1963 99). My partial reconstruction of this argument is as follows:

aDb	Premiss
$a = x$	Premiss
$b = y$	Premiss
xDy	Replacement property of equality
If xDy then . . .	(9)

Note that this is identical to the invalid form of argument (given above) concerning Able and Baker. One might want to get around this difficulty by assuming that all voters know all relevant facts of identity. However, this is a quite unrealistic assumption and is almost never fulfilled in practice. But if it were, we could let z be the best democratic choice and then always choose whatever alternative is

equal to *z* and thereby solve the social choice problem for democrats.

Let us look at the situation from another point of view. Suppose a teacher asked a student to create a predicate in some formal system of arithmetic which captures the informal notion of *is not equal to*. The student does so -- he calls the predicate "L" -- but it turns out that L is a transitive relation. Suppose he gave a proof of some purported theorem of arithmetic which made essential use of the property that L is transitive. Who would believe that such a proof helps us understand the informal notion of *is not equal to*? Similarly if Arrow attempts to have a predicate in his system which corresponds to the informal (and intensional) notion of preference, then the fact that in his translation the resulting predicate is extensional is sufficient to show that his proof is of doubtful use to anyone wishing to understand the informal notion of preference. Obviously the same comments hold true for indifference.

In sum, Arrow makes two fundamental errors which lead to an invalid argument: First, he neglects to consider the issue of self–reference (which is central to mathematical logic) and constitutional amendment (which is central to democracies). Second, he treats the intensional logic of preference as if it were an extensional logic of the sort familiar in mathematics. Arrow states that it was convenience which led him to "represent preference by a notation not customarily employed in economics, though familiar in mathematics and particularly in symbolic logic"(1950 331). But without restrictions on the replacement property of equality that representation is flawed. It is ironic that Arrow's use of symbolic logic probably resulted in making it harder to detect the logical errors in the proof.

7

Impossibility Proofs

What would constitute a good impossibility proof of the sort Arrow intended? By focusing on this question we can see how Arrow misunderstood what is needed to produce an impossibility proof for democratic social choice. There are four necessary steps in such a proof, which can be illustrated by comparing what Arrow attempted with what Alonzo Church achieved in his remarkable proof that it is impossible for there to be a decision procedure for arithmetic.

Step 1. *Propose a thesis.* Almost everyone is familiar with the intuitive idea of a recipe, or algorithm, or effective decision procedure. For example, there are an infinite number of problems of long division. Nevertheless the predicate *x is divisible by y* is solvable by using the recipe, or algorithm, or effective decision procedure, which we call long division. That is, although we can not solve all problems of division (there being an infinite number of them) we can solve in principle any division problem from this infinite class that is given to us, and do it by using the same method each time (long division). The question arises: is there an *exact* mathematical concept whose meaning captures the essence of what we mean in ordinary speech by recipe, or algorithm, or effective decision procedure? Church proposed the thesis that an exact mathematical concept -- that of a general recursive predicate -- explicates the ordinary intuitive concept of a decision procedure.[32] That is, all predicates of natural numbers (such as *x is divisible by y*) which possess a decision procedure (in the vague ordinary sense) are general recursive predicates (in an exact mathematical sense). Similarly all functions which we can effectively calculate (such as $x + y$) in the ordinary sense are general recursive functions in the mathematical sense.

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[32] Actually Church used a concept which is mathematically equivalent to the concept of a general recursive predicate.

The corresponding step in democratic theory is to find an exactly defined concept whose meaning captures the essence of what we ordinarily (and vaguely) mean by a democratic decision. I think it is fair to say that the four conditions on constitutions that Arrow proposes might plausibly be interpreted as an attempt at an implicit definition. The claim that this definition explicates the ordinary sense of a democratic decision might then be called "Arrow's thesis."[33] The burden of my argument in Chapters 2 through 5 is that these conditions do not constitute a reasonable explication of democratic social choice, that is, that Arrow's thesis is false.

Step 2. *Give evidence for the thesis.* Since Church's thesis is an empirical claim about our intuitions concerning algorithms, evidence must be marshaled for it, as for any other empirical claim. This evidence falls naturally into two parts: First, one must make an extensive historical study to determine what predicates of natural numbers were considered to have an algorithm in the past. These must be shown to be general recursive predicates. Second, one must try deliberately to construct a predicate which clearly has an algorithm but which is not general recursive. After showing that the most tempting means of construction fail, an open challenge is issued to anyone to produce a predicate with an algorithm in the intuitive sense, but which is not general recursive. The continuing failure to meet this challenge increases the likelihood of the truth of the thesis.

For democratic social decisions the evidence would likewise fall into two parts: First, historical evidence showing that actual democratic procedures satisfy the exact definition given by the social choice theorist. Second, hypothetical examples deliberately constructed both to be democratic and not to satisfy the conditions which define the constitution of the social choice mechanism. Arrow does not satisfy the first requirement. He states: "It's hard to imagine anyone quarreling either with the Pareto Principle or the condition of Non–Dictatorship. The principle of Collective Rationality may indeed be questioned. One might be prepared to allow that the choice from a given environment be dependent on the history of previous choices made in earlier environments, but I think many would find that situation unsatisfactory"(1967a 228).[34] Yet, as we have seen, it is easy

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[33] I first heard the term from Alan Taylor.

[34] Here again we see that Collective Rationality is questionable. For if the principle prevents a choice ever being dependent on previous choices and if, as we argued, it is incompatible with the use of a random selector, then how is a social decision to be made where the question is to

to quarrel with both the Pareto Principle and Nondictatorship on a number of grounds, not the least of which is historical evidence concerning democracies. Such empirical evidence is crucial. How impressed would we be with Church's theorem if it did not apply to decision procedures in analysis or topology or geometry or . . . ? Yet just this kind of indefinitely long sentence can easily be made about Arrow's theorem: It does not apply to democracies which use random selectors (such as Athenian democracy or most of our states), it does not apply to democracies which violate Transitivity (say, by using majority rule), it does not apply where fundamental political rights frustrate political desire (since these are incompatible with Preference Implies Choice and the Pareto Principle), it does not apply where there is unanimous objection to the Independence of Irrelevant Alternatives (such as could occur in an academic department), it does not apply . . . . Why am I obliged to believe that Arrow's theorem -- even if Arrow produces a valid proof for it --proves anything whatsoever about democracy? What is Arrow's theorem true of? *Detailed evidence must be provided to show its range of application.* The requirement of Step 2 explains why this monograph has so many references to democratic history and quotations from democratic leaders.

The second requirement concerning hypothetical examples Arrow does not attempt to satisfy at all. The examples given above in the critique of the Pareto Principle are alone enough to show that this requirement will not be satisfied. "Church's thesis," Stephen Kleene has observed, "would be disproved, if someone should describe a particular function, authenticated unmistakably as 'effectively calculable' by our intuitive concept, but demonstrably not general recursive"(1967 241). Arrow's thesis is likewise disproved since it is possible to describe a social choice mechanism (say, random choice selectors), authenticated unmistakably as democratic, but which demonstrably fails to satisfy the four conditions. But if Arrow's thesis is false then his theorem, even if valid, cannot have the significance he intended and many others accept.

Step 3. *Create a valid exact proof.* Let us look again at Church's theorem. The theorem states that the decision predicate *x is the Gödel number of a theorem of arithmetic* is not general recursive. The term *general recursive*, like *Euclidean circle*, is exact and Church's proof depends on this fact in a crucial way. Church's theorem thus requires no empirical evidence and, in this regard, is like the

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do or not to do some well-defined act and the vote turns out to be a tie?

Pythagorean theorem, or Euclid's proof that there are an infinite number of primes. Even if Church's thesis turned out to be false, Church's theorem would be unaffected (although our interpretation of it outside mathematics would be altered).

Arrow's theorem, however, commits logical errors involving self–reference and intensional contexts. As is, therefore, it proves nothing even if Arrow's thesis were correct. At this point it is worth noting another aspect of impossibility proofs, namely, that they are both difficult and profound. Consider tic–tack–toe. At a superficial level of understanding, a child views it as an open game: she wins some and loses some. Later, she develops a nonlosing strategy. Finally, we will suppose, she develops an argument which proves that it is impossible to find a winning strategy. This impossibility proof depends on profound knowledge, since the proof requires knowledge of every possible game. If the proof left out one case it would be worthless, for that case might be crucial to a winning strategy (which, until a proof exists, is an open possibility).

The situation is similar in mathematics. In geometry it is quite easy to bisect an arbitrary angle with a straightedge and a compass. In contrast, to show that it is impossible to trisect an arbitrary angle (with the same restrictions) requires a profound proof. Indeed it took at least 2500 years after mathematicians knew how to bisect an arbitrary angle before they could prove that it could not be trisected. The latter proof depends on a precise definition of *all possible constructions with a straightedge and a compass*. No matter how complicated, all constructions must be included in the definition. If any construction is left out, an impossibility proof depending on that definition would be fallacious. An impossibility proof is a type of logical balloon: any gap brings total collapse. Likewise if there is any gap in Arrow's definition of the democratic social choice mechanism (such as leaving out democratic random devices) his proof is refuted although, of course, it is conceivable that another proof to the same effect might be constructed.

Step 4. *Draw the impossibility conclusion.* By Church's thesis every predicate with an algorithm is general recursive. By Church's theorem the decision predicate *x is the Gödel number of a theorem of arithmetic* is not general recursive. Hence it is reasonable to conclude that there is no algorithm for the decision predicate for arithmetic. In other words, the decision predicate is not solvable in the ordinary sense and thus arithmetic has no decision procedure. Notice that this conclusion depends on the truth of both Church's thesis and Church's theorem, and thus on the quality and quantity of evidence for the thesis,

and the validity of the proof of the theorem. Without the preceding steps, the fourth step cannot be legitimately taken.

Unfortunately a host of writers have been all too willing to "draw the impossibility conclusion" with regard to democracy. Samuelson is typical: "Arrow asks: . . . how can we define a social preference ordering. . . that obeys a few appealing axioms? . . . He then proves by elegant reasoning that it would involve a self–contradiction for there to be a solution satisfying all of these appealing axioms. Aristotle must be turning over in his grave. The theory of democracy can never be the same . . . since Arrow"(1966 4:935). Despite Samuelson, a "perfect democracy" has not been shown to be a "logical self–contradiction"(1966 4:938); nor, despite Wolff, has Arrow shown that inconsistency "infects virtually every method of social choice which can lay a reasonable claim to being called 'democratic' "(1970 63). However, ideas have consequences and these conclusions can diminish commitment to democratic institutions and contribute to the belief that democracies are incapable of excellence in solving social problems. At any rate, this refutation is motivated by a desire to defend democracy and those moral ideals without which it is not justifiable to call a government "democratic."

It may be, by the way, that, if we ever get an adequate exact definition of democracy, some impossibility proof or other might legitimately be proved. I believe that Arrow was thus not at all misguided in his intention but was premature since at this point we can not know what such a proof might look like. We do not have enough insight into ourselves or democracy.

Finally, it should be noted that inconsistency in a political constitution is not like inconsistency in a formal system based on classical symbolic logic. In the latter case the system is rendered useless but an inconsistent political constitution can be quite useful. As we have seen, the American Constitution is inconsistent on the issue of whether a state, without its consent, can be deprived of "equal suffrage" in the Senate. Yet that inconsistency has for more than two hundred years been harmless. In fact, it might be argued that some contradictions should be built into a political constitution so that it gains the most widespread support. Contradictions, when they arise in practice, can then be taken care of piecemeal by judicial interpretation or subsequent legislation. Here I agree with Arrow: an inconsistency in the constitution of a system of social choice is "embarrassing" and one should try to eliminate it or, if that is not possible, render it harmless. My concern is to show that Arrow's theorem does not imply that striving for a

practical and excellent democratic constitution for social choice is somehow irrational.[35] I believe that Arrow's theorem is as subject to misuse as the allegedly scientific claims that one race or sex is mentally inferior to the other. Perhaps someday a would–be tyrant will appeal to it by arguing that dictatorship is consistent while democracy is not, and that therefore the abandonment of democracy is only reasonable. In general, Arrow's theorem certainly can add weight to the belittlement of democracy and can thus be used to weaken commitment to democratic institutions. In my judgment this potential effect of the theorem is not too remote to be worth worrying about. It is for this reason that I have been so unrelenting in my criticisms. I hope to stimulate others to correct possible deficiencies in this monograph by, for example, publishing objections to the proof which I may have overlooked or misstated.

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[35] Arrow himself suggests continued striving to overcome the difficulties: "The philosophical and distributive implications of the paradox of social choice are still not clear . . . . I hope that others will take this paradox as a challenge rather than as a discouraging barrier." (1974a 271)

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Philosophical Presuppositions of Arrow's Proof

We will now examine the philosophical foundation on which Arrow bases his proof. Arrow rejects the Platonic conception that the social good can be defined without reference to the desires of the individuals making up society. Instead he accepts Bentham's utilitarianism. The aim of this doctrine, according to Arrow, was to "ground the social good on the good of individuals" and "the social good was in some sense to be a composite of the desires of individuals"(1963 22). An important question to raise to this sort of view is: Are all desires equal? We normally make a distinction between "long–term" or "sober" or "rational" desires and "short–term" or "frivolous" or "irrational" ones. For example, the person who says, "If I drink at the party tonight, do not let me drive home" and then, after drinking, insists "I want to drive home" is expressing two desires, but are they equally worthy of satisfaction? A community that expresses an interest in justice but later forms a lynch mob is likewise expressing two social desires. A democracy cannot treat these desires as equally worthy of being the social choice.

If we ask what are the most long–term or sober or rational desires a democratic society can have, the answers in the Declaration of Independence come to mind: life, liberty and the pursuit of happiness. The Founding Fathers thought these desires were universal to all human beings and thus natural. Consequently, they believed the corresponding rights could not legitimately be alienated from the people. Bentham, in contrast, sharply criticized the Declaration in a letter written in September of 1776. He wrote: "This they 'hold to be' a . . . 'truth self–evident'. At the same time to secure these rights they are satisfied . . . that Governments should be instituted. They see not, or will not seem to see that nothing that was ever called government ever was or ever could be in any instance exercised save at the expence of one or other of those rights. . . . In these tenets they have out done the utmost extravagance of /all former/ fanatics . . . "(1968 1:341–3). Elsewhere Bentham says the Declaration "presented itself to my conception from

the first, as what it has always continued to be, a hodge–podge of confusion and absurdity . . . "(1859 10:63; see also 10:57). One wonders whether Arrow would defend the Declaration, or the Nuremberg trials. I certainly admire his commitment to the causes of equality and justice which is evident from his popular writings and in numerous biographical comments about him.[36] Yet the moral ideals represented by fundamental democratic rights play no part in his constitution for social choice. Hence one wonders where, in his philosophy of social choice, crimes against humanity fit in. Were the Nuremberg trials based on a philosophical error?

In Chapter 6 we saw that a self–referential argument showed the invalidity of Arrow's argument for his theorem. But a self–referential argument does more than that. It raises, in political constitutions, fundamental logical problems concerning meaning and mutability which are very difficult to solve. The Founding Fathers made direct appeal to unchangeable and unalienable rights (created by Nature or God) as the moral foundation of the American republic. They have often been criticized for not questioning this foundation, but it had the effect of preventing moral nihilism. I believe the foundation should be questioned but it is one thing to question it, it is quite another to propose a system of democratic social choice which has no moral foundation whatsoever. Arrow treats the assumption of moral values which serve as the foundation for democratic social choice as a kind of ignorant prejudice of ages previous to ours which only soft–headed philosophers would suggest today. But even within the realm of pure mathematics "realist" and "Platonic" claims are made by outstanding logicians, such as Frege, Church or Gödel.[37] A formalist such as Kleene can also write, " . . . if we are not to adopt a mathematical nihilism, formally axiomatized mathematics must not be the whole of mathematics. At some place there must be meaning, truth and falsity"(1967 193). Similarly formalized and democratic social choice theory must at some point have ethical content. Otherwise we have the moral nihilism which was discussed in relation to the Pareto Principle.

Democracy cannot be built on an ethically neutral foundation. Arrow states that his "interpretation of the social choice problem agrees fully with that given by

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[36] See the bibliography of Arrow's nontechnical articles in Feiwel 1987 741. This volume and its companion (Feiwel 1987a) contain extensive biographical information on Arrow.

[37] Arrow himself is apparently a Platonist with respect to mathematical objects. See Arrow 1951 130.

[Abram] Bergson"(1967 14). Arrow is referring to the passage where Bergson says: ". . . the problem is to counsel not citizens generally but public officials. Furthermore, the values to be taken as data are not those which might guide the official if he were a private citizen. The official is envisaged instead as more or less neutral ethically. His one aim in life is to implement the values of other citizens as given by some rule of collective decision–making"(Bergson 1954 242). But how far can one push this point of view? Provided only that the rule of collective social choice requires it, is the democratic public official as ready to declare an aggressive genocidal war against a foreign people, as a peaceful war against domestic poverty? Let us look at another example. Imagine two countries A and B each with some mildly antiSemitic laws. Let us suppose that neither contains any Jews, but that many members of each country fear that Jews will be immigrating and the antiSemitic laws are meant to discourage this. Now, due to some international incident, suppose that both countries experience a strong rise in antiSemitic feeling. The social decision in country A is to pass more and stronger antiSemitic laws, that of country B is to repeal the ones it already has. We might think country B's institutions are praiseworthy to bring about such a result, especially under the circumstances. Arrow's social choice principles would appear to commit him to an opposite conclusion. "Since we are trying to describe social welfare," he says, "and not some sort of illfare, we must assume that the social welfare function is such that the social ordering responds positively to alterations in individual values, or at least not negatively"(1950 336).[38] In sharp contrast, our Constitution, instituted in part to "promote the general welfare," created a government specifically designed to respond negatively to any combination of individual preferences which leads to the violation of rights. Government officials were not to be "more or less neutral ethically" nor was the social good understood merely to be a "composite of the desires of individuals." Madison's minutes of the Constitutional Convention, as well as the *Federalist Papers*, make this abundantly clear.

By his failure to pay attention to the quality of preferences Arrow is forced to fall back on a method of social decision whose moral bankruptcy was exposed

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[38] It is completely unbelievable that Arrow would actually act the way his words suggest. Would Arrow, if he were an advisor to a public official who was "more or less neutral ethically," really try to devise social choice mechanisms to make country B more like country A? I am critical of Arrow's conception of social choice theory, not of Arrow, the man who defends human rights.

hundreds of years ago. These are the words of Alexander Hamilton from June 1788: "Experience has proved, that no position in politics is more false than this [that is, that a pure democracy would be the most perfect form of government]. The ancient democracies, in which the people themselves deliberated, never possessed one feature of good government. Their very character was tyranny; their figure deformity; When they assembled, the field of debate presented an ungovernable mob, not only incapable of deliberation, but prepared for every enormity"(1961 5:38–9).[39] Madison makes essentially the same point: "In all very numerous assemblies, of whatever characters composed, passion never fails to wrest the sceptre from reason. Had every Athenian citizen been a Socrates, every Athenian assembly would still have been a mob"(1962 10:505). Even if Arrow's proof is sound the fact that at best it applies only to a form of democracy which already is known to have fatal flaws certainly diminishes its significance. (Would we be impressed if someone took Hobbes' proof that the circle can be squared -- a proof whose invalidity was repeatedly exposed while Hobbes was still alive -- and then, after translating it into the language of symbolic logic, finds an error that no one had noticed before?) Here we see there is something anachronistic about the theorem. Yet it gets its fame from a supposed important connection with contemporary democratic theory and practice.

Any successful defense of the theorem would have to satisfy three

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[39] Why was the negative view of democracy so common in the eighteenth century? Mogens Herman Hansen gives the following answer: "Probably because Roman literature was studied much more intensively than the Greek orators and historians, and the Greek author mostly read by those who were active in the American and French revolutions was Plutarch, whose pejorative view of Athenian democracy is all too apparent in the *Life of Phokion* and only half concealed in the Life of *Demosthenes*. . . . The principal advocates of liberty in the modern world did not find their beliefs supported by ancient authors -- probably because they studied the philosophers instead of reading the orators"(1989 26). Nevertheless, it is clear that Hamilton's objection is to democracy without built-in safeguards, not to democracy with those safeguards. As he put it in 1777: "But a representative democracy, where the right of election is well secured and regulated & the exercise of the legislative, executive and judiciary authorities, is vested in select persons, chosen *really* and not *nominally* by the people, will in my opinion be most likely to be happy, regular and durable"(1961 1:255; this passage is quoted in part by Hansen (33)). Representation was also the important qualification for Jefferson as a champion of democracy. The ancient Greeks, Jefferson said, "knew no medium between a democracy (the only pure republic, but impracticable beyond the limits of a town) and an abandonment of themselves to an aristocracy, or a tyranny independent of the people. . . . The full experiment of a government democratical, but representative, was and is still reserved for us. . . . The introduction of this new principle of representative democracy has rendered useless almost everything written before on the structure of government . . ."(1903 15:65–66).

conditions: First, the question of validity would have to be faced. Do Arrow's works contain a valid proof *which needs no additions or qualifications*? If so, then objections to its validity of the sort raised in this monograph should be answered. If not, a valid proof has to be produced. This by itself is not particularly significant. After all one can take virtually any proof and make it valid by changing the axioms or conclusion.[40] The new theorem and proof must be shown to be close enough to the old to justify calling it "Arrow's theorem" or "Arrow's proof." This brings us to the second condition: The valid proof must apply to democracy as currently understood. A defense must show that the range of the new proof's applicability is not so narrow that it ceases to have any significance for contemporary democratic theory or practice. For otherwise the defense is itself a kind of refutation: the reconstructed proof is valid but is irrelevant to democratic social choice. Finally, the defense must show that the theorem not only applies to democracy but tells us something important. If its significance is reduced to the claim that, without tie–breaking techniques, ties lead to democratic indecision, or that the order of voting can alter the outcome of elections, or that a majority can be against every choice when at least three choices are available, nothing new has been shown although the argument might now be tighter than before. Again, such a defense would be a kind of refutation: the argument would be shown to be sound but since the result would be unimportant all the claims that Arrow's theorem is somehow profound would be false.

Arrow's philosophical comments show, I think, that he misunderstands the democratic spirit. Let me use an analogy. Imagine that you bought a ticket to a professional football game. You watch the game and immediately find out that each team decides not to oppose the other. Nevertheless they play according to the rules. Each kickoff results in a touchdown. Such play continues until a regulation game has been completed. Imagine that, feeling swindled, you demand your money back. Would you be impressed by the reply: "This has been a model game. All the rules were followed and there were no penalties for unsportsmanlike conduct, or anything else." Of course not. The point is that football presupposes

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[40] An invalid proof showing that an arbitrary angle can be trisected using just a straightedge and a compass can be modified to show (correctly) that it is possible to trisect a right angle by those means. This modification sheds no respectability on the invalid proof. Notice too that mathematicians do not honor any of the many invalid proofs that the circle can be squared using a straightedge and a compass even though it has recently been shown that it is possible to square the circle in another sense of that expression. See Gardner and Wagon 1989.

a certain competitive spirit without which it becomes meaningless even if the normal rules are followed.

Similarly democracy must be informed by a humane spirit without which it becomes meaningless even if procedures normally associated with democracies are followed. Arrow maintains he is "seeking to model democracy"(1977 614). Yet since he ignores the moral reasons for seeking a democracy in the first place, he produces an absurd caricature. This is seen most clearly when he proposes some alternative assumptions. One of these is called Nonimposition, according to which "every social choice is in fact permitted by the social choice procedure"(1977 617).[41] This principle is consistent with the execution of children, the decision of one generation not to educate the next, or the promotion of an aggressive genocidal war. No government could be considered democratic which did such things, any more than a game could be considered football where the opposing teams cooperated. It would not matter if the government followed procedures normally associated with democracies or if the rules of football were obeyed. The point is this: Governments have a *raison d'être*. In the case of democracy this means endeavoring to fulfill certain moral objectives. The outcome of any procedures of social choice can not be democratic if they undermine the democratic ideals which serve as the justification for having those procedures in the first place. The murder of a person by a lynch mob is not an example of a democratic practice even if there was a unanimous vote beforehand. The claim that "every social choice is in fact permitted by the social choice procedure" is not

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[41] The Principle of Nonimposition is an explicitly stated assumption in Arrow's first (1950 338), and in his most recent, version (1987 125) of the theorem. Notice that there is something paradoxical about this principle for if it is incapable of change by social choice, it is self–refuting. After all, according to Arrow his conditions are "value judgments and could be called into question"(1950 339). On the other hand, if it can be changed it is ineffectual. The people need only make the social choice to repeal this principle and then modify the social choice mechanism so that it imposes any set of values they want. More is being attempted here than one can consistently do -- at least the problems of mutability make a correct statement of the principle very difficult. Any adequate social choice mechanism should contain a device for amending itself *à la* the American Constitution. The logic is tricky (see Suber 1990) but is, I think, analogous to that in artificial intelligence programs which must be designed to rewrite themselves in light of their "experience." Thomas Hobbes called the state an "Artificiall Man". The great problem for democratic social choice theorists is to give this "Artificiall Man" an artificial intelligence which is invariably intent on securing fundamental democratic rights.

a description of democracy.[42] An ethically neutral democracy is a charade. "I regard utility as the ultimate appeal on all ethical questions," John Stuart Mill once said, "but it must be utility in the largest sense, grounded on the permanent interests of man as a progressive being"(1978 10). Emphatically the Pareto Principle and the Principle of Nonimposition are not "grounded on the permanent interests of man as a progressive being." Would it not be a worthy goal for democrats to devise a constitution for social choice which had this kind of grounding?

When discussing homosexuality Arrow states: "Personally, my values are such that I am willing to go very far indeed in the direction of respect for the means by which others choose to derive their satisfactions"(1967a 223). But in other cases one wonders, how far? One doubts it would include those who get their satisfactions from torturing dogs or children. "It is better," as Mill beautifully put it, "to be a human being dissatisfied than a pig satisfied; better to be Socrates dissatisfied than a fool satisfied"(1948 9). Democracies ought not to satisfy those who desire to violate other's rights. Arrow is enmeshed in a dilemma: if he goes all the way "in the direction of respect" he is a moral nihilist; if he draws a line and recommends imposing certain values on the social choice mechanism to the public official, who is "more or less neutral ethically," then he must admit that that advice (according to his social choice principles) is illegitimate since it is based on nothing but his personal distaste for having dogs or children tortured, and not on what might be the public's desire at the moment.

Liberal democratic thought has traditionally made a distinction between those preferences which are, and those which are not, worthy to influence social policy. Thus Jefferson says: "The legitimate powers of government extend to such acts only as are injurious to others. But it does me no injury for my neighbor to say there are twenty gods, or no god. It neither picks my pocket nor breaks my leg"(1972 159). Arrow, however, rejects the traditional liberal distinction:

> The only rational defense of what may be termed a liberal position, or perhaps more precisely a principle of limited social preference, is that it is

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[42] Perhaps it is worth recalling our earlier analogy to medicine. Murders committed by medical teams -- say, in concentration camps -- are not part of the practice of medicine, even if medical technology is used exclusively. "Every medical choice is in fact permitted by medical procedures" is not a description of medicine.

> itself a value judgment. In other words, an individual may have as part of his value structure precisely that he does not think it proper to influence consequences outside a limited realm. This is a perfectly coherent position, but I find it difficult to insist that this judgment is of such overriding importance that it outweighs all other considerations. (1967 12)

Mill, in sharp contrast, does find it of "overriding importance." "All that makes existence valuable to anyone," Mill tells us, "depends on the enforcement of restraints upon the action of other people"(1978 5). American democracy was built on "a principle of limited social preference." How else can we understand the Declaration of Independence or the Constitution? Arrow says ". . . I want to emphasize that value judgments in favour of limited social preference . . . must be counted as part of the value systems which individuals use in the judgment of alternative social actions"(1967a 223). Of course. But those who follow liberal political principles would add, as Arrow does not, "value judgments in favor of unlimited social preference which lead to the violation of individual rights must not be counted as part of the value systems which can influence social policy."

In sum, a detailed critique of the kind of democracy that Arrow proposes can be found in the writings of the Founding Fathers. Some of these criticisms even then were many centuries old. Arrow attempts to devise a neutral social choice procedure which responds to individuals' desires regardless of the content of the desires, or how they were formed. The result is morally disastrous. John Adams once declared: "None but an idiot or a madman ever built a government upon a disinterested principle"(Haraszti 220). An exaggerated statement, perhaps, but thinking about it might make us reflect that the technical aspects of social choice theory might, if applied thoughtlessly or maliciously, be just as heartless or dangerous as the technology of modern science when it is applied thoughtlessly or maliciously.

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The Influence of the Theorem

D. H. Monro has pointed out that Bentham, in order to save his system, was forced to postulate moral rights and moral duties in senses different from those which his own philosophical pronouncements would allow (1967 284–5). It seems to me that Arrow follows Bentham in this, as he does in so much else. At any rate, the constitution Arrow gives for democratic social choice presupposes hedonism and behaviorism but, when considering other topics such as justice or taxation, he recognizes realities which are inconsistent with those presuppositions. Thus when he is discussing the "distributive implications of social choice" Arrow states: ". . . each individual may be assumed to have a preference ordering over all possible social states. This ordering expresses not only his desire for his own consumption but also social attitudes, his views on justice in distribution or on benefits to others from collective decisions"(1985 142)[43]. But when discussing his theorem he gives a different answer. Responding to a question of Richard Brandt as to whether he is dealing with choices motivated by personal welfare or by moral principle Arrow states that ". . . strictly speaking, the preference scales that I deal with are those relating to personal welfare only. The range of moral obligations is part of the social decision to be arrived at by the collective decision–making machinery"(1967 119).

Again, when discussing Rawls's *Theory of Justice* he states: "A political system in which there is no other–regardingness will not function at all. . . . I would hold that the notion of voting according to one's own beliefs and then submitting to the will of the majority represents a recognition of the essential autonomy and freedom of others. It recognizes that justice is a pooling of irreducibly different individuals, not the carrying out of policies already known in

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[43] Elsewhere Arrow says that even our "economic life depends for its viability on a certain limited degree of ethical commitment. Purely selfish behavior of individuals is really incompatible with any kind of settled economic life" (1973a 314).

advance"(1973b 258). Yet Arrow denies that even a strong belief in democracy would be sufficient to solve the problem of social welfare since that "would require that individuals ascribe an incommensurably greater value to the process than to the decisions reached under it, a proposition which hardly seems like a credible representation of the psychology of most individuals in a social situation"(1963 89–91).[44] One wonders how Arrow reconciles this hedonistic psychology with a "recognition of the essential autonomy and freedom of others"?

Finally, when discussing taxation Arrow makes the following quite reasonable pronouncement: "I think we may safely agree that the notion of democracy has two components, both indispensable: 1) the securing of the freedom of the individual so that he may develop his individual potential; 2) a symmetric mutual respect of the individuals in the society for each other"(1974 23). Yet these "indispensable components" play no part whatsoever in Arrow's constitution for democratic choice. It is thus not surprising that Arrow's theorem is irrelevant to actual social choices. This point was make years ago by Gordon Tullock: ". . . as far as I can see, almost all students in the field, including Arrow himself, simply disregarded his proof when they turned to policy matters. Policy recommendations tended to be based on much the same reasoning as that of standard economists, with public choice having very little effect"(1979 27).

Here again it seems to me that Arrow is in a dilemma: if in the end he accepts a hedonistic and behavioristic view then many of the realities which he recognizes become difficult to explain. Thus Arrow states: "To some extent, certainly, the voter is cast in a role in which he feels some obligation to consider the social good, not just his own. It is in fact somewhat hard to explain otherwise why an individual votes at all in a large election, since the probability that his vote will be decisive is so negligible"(1969 61). Why make the distinction between the voter's own good and the social good if the voter will not or cannot or should not consider anything but his own good? And what is Arrow's explanation of why people vote in large elections (given that hedonism is true)? On the other hand, if he rejects hedonism and behaviorism how can his proof be sound? For instance, if a community unanimously prefers x over y but the social choice becomes y out of consideration for the individuals in a neighboring community, what then

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[44] To me, at any rate, American elections -- which are often bitterly fought and then the outcome peacefully accepted -- provide a good counterexample to this claim about the "psychology of most individuals."

becomes of Preference Implies Choice, the Pareto Principle, and the Independence of Irrelevant Alternatives? The irrelevance of the Arrovian social choice theory to democracy is not due to the technical apparatus but its flawed philosophical assumptions, that is, its technical apparatus is based on an amoral and behavioristic conception of human choice, while democracies are not.

In my view Arrow's separation of social choice theory from actual democratic social choices has been a fateful blunder.[45] The consequences of this blunder can be judged by comparing it with computer science. The modern theory of computers may be dated from Alan Turing's paper in 1936. A dozen years later both Duncan Black and Arrow began their work in social choice theory. Yet what a difference there has been in the results! The theory and technology of computers have influenced nearly every academic discipline, it has stimulated our imagination, transformed life in developed countries, made possible feats otherwise unthinkable (such as space exploration) and tremendously lowered the costs of production, information and communication. With only a small amount of exaggeration we can say that social choice theory is a fascinating academic game but otherwise has been unimportant in human culture.

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To see what might have been let us imagine that its development had paralleled that of computers and that we have come across an annotated bibliography of important articles and books written on social choice in the last forty years. Omitting existing works and everything else except the titles and commentary, my fantasy is that the bibliography might have looked something like the following:

"The Metatheory of Democratic Social Choice" -- This article proposed a metatheoretical program for social choice corresponding to Hilbert's program for mathematics. (Hilbert's program required that each area of mathematics be shown to be consistent, correct, complete and to possess a decision procedure.) It is the first work in social choice theory to confront directly the fact that some inconsistent constitutions (unlike inconsistent formal systems) are useful. The

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[45] I do not want to maintain that this separation applies universally. The theory of approval voting (noted in Footnote 9) is an excellent example of social choice theory which is directly relevant to actual democratic choices.

author uses a nonstandard logic in which a contradiction does not imply every proposition. (For an example of such a logic, see Anderson and Belnap 1975) After this paper every serious investigator has agreed that we need dynamical computer models to understand democracy and that both the traditional political analysis and "formal system social choice theory" must in the end be inadequate.

"Unsolved Problems in Social Choice Theory" -- This article has been one of the great boons to technical research. The author surveys the social choice problems of humankind and proposes -- *à la* Hilbert -- twenty-three unsolved technical problems which, if solved, would be of the greatest use to humanity. Some of the problems have subsequently been solved but it will probably take another fifty years to solve them all.

"Turing Machines and a Social Choice Selector" -- This work explores the question of whether there can be a universal social choice selector (corresponding to a universal Turing machine) which could make a reasonable social choice in any circumstance in which a reasonable social choice could be obtained by any method. Since the problems of humankind are to a large degree due to bad social choices the importance of the question is hard to overestimate. This work stimulated the creation of social choice science fiction, the way Turing's 1950 paper "Computing Machinery and Intelligence" helped to inspire *2001*. It is very hard to conceive the enormous benefits of a truly excellent social choice mechanism but it was the fiction inspired by this essay which stimulated our imagination and proved to be a decisive influence in the public's generous support of social choice research.

"The Technology of Social Choice" -- This paper began the social choice industry; it attempted to give computers a social intelligence. What is remarkable about the author is that she not only understood the theory of social choice but that she also saw the commercial possibilities of social choice technology. As a result, the author's company is now trying to create, for a well-known large corporation, a dynamical computer model of the values of shareholders, the employees, the public, against the background facts of demography, laws, competition, etc., and to automate the best choices for management to make. The outcome of this particular project is still in doubt but it is safe to say there is nothing that is more carefully watched by the commercial world.

"Self-Referential Problems in Social Choice Theory and Practice" -- At once playful and tantalizing, this paper has two main parts which do not fit together very well. In the first, the author proposes an extensive statement of the professional ethics of social choice theorists. He then asks what a political

constitution would have to be like in which the profession of social choice theory itself could flourish. In the second, the author looks at paradoxes of self–reference such as the paradox of self–amendment. The paper has been the fountainhead of a large number of papers on self–referential paradoxes and the analysis –– from a social choice perspective –– of such notions as national self–consciousness and national self–deception.

Human Nature and Social Choice Theory –– This book revolutionized our conception of social choice engineering. As the author puts it, the mechanisms of social choice engineering need not be run by people of "system"[46] but instead can be designed to promote the autonomy and freedom of the individual. The author sets herself the following problem: In the eighteenth century the Founding Fathers tried to take into account everything that was known of human nature from literature, history, political science, and philosophy in the construction of the American Constitution. Here the author sets out a program to do the same thing using such disciplines as biology, psychology, sociology, anthropology, history, philosophy, and so on. The book is more than a decade old and the eventual success of the program is still in doubt. However, it has forever put to an end the arm–chair psychologizing about human nature that was so common in the early years of social choice theory. No one suggests anymore that humans are selfish. Instead they look to see how the people define "selfish," when they are selfish, in what ways they are selfish, in what environments they are less likely to be selfish, how the social choice structure can be cleverly designed to overcome the specific kinds of selfishness which might lead to social choices which undermine democratic ideals, and so forth. The book also introduces the idea of a friendly law, that is, one that is easily understandable, on the analogy of a friendly computer program. Billions of dollars have now gone into the project of making U. S. laws friendly but the payoff in terms of compliance and respect for the law has made the price seem very cheap indeed.

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[46] The reference is the following quotation of Adam Smith: "The man of system. . . is apt to be very wise in his own conceit; and is so enamoured with the supposed beauty of his own ideal plan of government, that he cannot suffer the smallest deviation from any part of it. . . . He seems to imagine that he can arrange the different members of a great society with as much ease as the hand arranges the different pieces upon a chess–board. He does not consider that the pieces upon the chess–board have no other principle of motion besides that which the hand impresses upon them; but that, in the great chess–board of human society, every single piece has a principle of motion of its own, altogether different from that which the legislature might chuse to impress upon it" (1982 233–4).

"The Social Choice of Self–Protective Devices" -- This article by a well–known economist is a survey of the most efficient social choice devices. By "efficient" the author means those devices or institutions which will, first, utterly frustrate attempts to invade rights or violate the moral ideals of democracy and second, not contribute to democratic paralysis and third, actually do the will of the people at the least possible cost (as judged by the people). In the course of her analysis the author considers the maximum extent that random devices can reasonably be used to promote the goal of democratic equality since she believes these are the least susceptible of manipulation. Of special interest is the extraordinarily clever way in which computer modelling is used to test various self–paternalistic suggestions. The heart of the paper reverses the emphasis of Madison's well–known statement, "Had every Athenian citizen been a Socrates, every Athenian assembly would still have been a mob"(1962 10:505). After extensive empirical interviews with, and observation of, corrupt officials she asks: "If every legislator is corrupt in the same ways that legislators have been corrupt in the past, what social choice mechanisms would lead them to pass only wise and just laws?" In spite of the seeming unreality of the question the discussion is extremely suggestive.

Critical Reason, Liberal Education and Social Choice Theory -- This book takes as its point of departure Madison's conception of the "cool and deliberate sense of the community." What good, the author wants to know, is knowing that it is the cool and deliberate sense of the people that is the final political authority if we have no means for finding out what that sense is? He believes that we have some idea of Madison's phrase in criminal law: a representative sample of the community is chosen (called a jury), the jury is made knowledgeable (that is the purpose of a trial), and then deliberately decides whether or not the accused is guilty. The author generalizes this basic idea to other social issues, that is, he tries to devise practical mechanisms and institutions which would accurately answer the question: If the people were made as knowledgeably on some social issue as possible, if they gave it thoughtful and careful consideration, what would they say? There are a number of interesting suggestions made although it would take a generation of work to carry out his basic idea. His last chapter is on the idea of a liberal arts education, that is, an education which would prepare the people for participation in the institutions which operationally define the "cool and deliberate sense of the people."

"The Social Choice of Civic Virtue" -- "What social choice arrangements

would tend to promote civic virtues?" So begins this interesting article which builds on the preceding work. The author conducted many experiments in which a representative sample of the people coolly and deliberately picks the virtues which they would like their leaders to exemplify. Using a variety of methods she found individuals who best exemplify those virtues (as judged by the people). After extensive interviews and experiments the author considers how campaigning and elections could be structured so that this set of persons would be most tempted to run, and if they ran, would be most likely to be elected. This article inspired the formation of a new journal, *Civic Virtue and Social Choice,* dedicated to designing democratic techniques of social choice so as to improve the quality of political candidates.

On Giving Social Choice Theory a Social Conscience -- In 1787 the small states considered themselves vulnerable and the social choice institution which we call the Senate was created to protect them. It has been wonderfully successful in this particular aim. This book applies social choice theory to determine the best social choice devices to protect vulnerable individuals -- such as those who are both poor and uneducated -- so that they actually have an influence in public affairs proportionate to their numbers and their plight. Of particular note is the consideration of tricameral legislatures (whose members are chosen by three different schemes). One of these legislatures is specifically designed to represent the interests of the vulnerable individuals. Computer modelling shows that tricameral legislatures better represent the views of the people and actually reduce democratic paralysis provided that only two of the three houses must pass a bill for it to become law (assuming the President approves).

"The Strategic Democratic Initiative for World Peace" -- The author's premiss that democracies do not go to war with one another is exaggerated, but not by very much. He considers how far social choice mechanisms can go in undercutting the antidemocratic feelings of a country. Suppose, for example, the United States could impose government with democratic forms on a population in which many oppose democracy and distrust and hatred is widespread. The author proposes democratic social choice mechanisms which he claims, once instituted, would be extremely hard to change into antidemocratic mechanisms and would likely lead to a new generation of committed democrats. An analysis is made of the imposition of democratic forms on Germany and Japan after World War II after which the author asks: How could such imposition best be done in a variety of circumstances?

Ancient Greece and Social Choice Theory -- This book set new standards of empiricism in social choice theory. Using information retrieval technology, the author examined every extant ancient use of Greek terms for democracy and related concepts. He then devised a set of assumptions which implicitly defined *ancient Greek democracy* and which, although relatively simple, fits every use. It is hard to imagine greater meticulousness in the employment of empirical evidence. What the author definitively established is that there is a single core conception which underlies seemingly contradictory uses. There are now proposals underway to duplicate his work using the English word *democracy*. Even though such a project would be very difficult and expensive most social choice theorists think the time and effort would be well repaid in new insights.

South Africa and Social Choice Theory -- One of the great landmarks of humane scholarship. It was this work which eventually led to the writing of a new constitution for South Africa. Recent events suggest that it is likely that the South African government will adopt it. The story can be briefly told. The authors proposed that a new constitution be written satisfying the highest empirical and theoretical standards of social choice theory. Money was raised not only from individuals but from many governments who were reluctant to use economic sanctions but were willing to make cash contributions. Well over a billion dollars was raised and there was much donated time by individuals and organizations. The authors organized an army of logicians, computer scientists, social choice theorists, mathematicians, philosophers, sociologists, historians, economists, and so forth and created an enormously complex and dynamic ethical–political model of the country. Millions of hours of computer time have shown that the life prospects of twenty–five representative South Africans -- all very different from one another -- will each, under the new constitution, be much higher than under the old. Of particular interest is the construction of simple devices which recognize racism and distrust -- such as having only multiracial juries -- but which over time will self–destruct to produce a society relatively free of racism. This is the first place to look for those who want to understand how the imaginative use of social choice devices can undermine racism.

"Social Choice and Democratic Revolution" -- In America the end of the eighteenth century was a great age of writing democratic constitutions. This process attracted the greatest minds and talents of America. Here again the focus is on the American Constitution and the author considers how we can take all that is known and produce a simple and elegant constitution for America which

perfectly embodies the informed values of the American people. As in the work of Kepler or Einstein aesthetic considerations and perfectionism dominate this essay. So great are the benefits of a political system which is designed exactly to fit the informed values and considered judgments of the people that there is now a world–wide race for leadership in social choice technology. As a result of this work another great age of constitution writing may be upon us.

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So much for my somewhat playful fantasies of what might have been. I believe social choice theory could effectively be applied to problems such as these. There already is a substantial body of technical results which could be made relevant and some places where it has been made so. My attack on Arrow's theorem is by no means meant to be an attack on everything in the discipline itself.[47] On the contrary, it is just because there is here an impressive body of knowledge -- and many opportunities for action -- that it is especially regrettable to see how little effort there has been to improve actual democratic choice mechanisms. With respect to computers I think it is fair to say that the pace of progress is such that there is little or no gap between what is possible in theory -- given current technology -- and what is actually done, whereas in social choice there is a huge gap between possibilities and performance. Why is not a useful social choice technology being developed and implemented as fast as could be reasonably expected? There are no doubt numerous factors but a principal one may be that many believe that Arrow's theorem shows that the search for truly practical social choice mechanisms is like the search for perpetual motion machines -- only the ignorant would attempt it.

It is thus not surprising that serious attempts to make our laws consistent are largely neglected. Let me give two examples. According to Sections 1 and 2 of Article VII of the Constitution of Utah if there is a tie vote for Governor, Lieutenant Governor, State Auditor, State Treasurer, or Attorney General "the two houses of the Legislature, at its next session, shall elect forthwith by joint ballot

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[47] Let me say again that I have deliberately confined my criticisms to Arrow even where it might have been easy or appropriate to comment on others. This seems to me to be reasonable for, if my contentions about the theorem turn out to be substantially wrong, why compound the errors by applying them to other writers? On the other hand, it they are substantially correct, that possibility can not be confirmed until it is so judged by a community of scholars.

one of such persons for said office"(UEL 1990 10). This directly contradicts the subsequent Utah code which states that these offices, in case of a tie, shall be determined by lot.[48] Such contradictions are not hard to find. For example, some seventh–grade students in Arizona discovered one in their state constitution and, in February 1988, pointed out to an Arizona house committee that Article V, § 2 of the constitution allows only a "male person" to be governor (among other offices). This not only contradicts federal law but is inconsistent with other provisions of the Arizona constitution itself (Article 2 § 13 and Article 7 § 2).[49] Developing social choice mechanisms which systematically eliminate such contradictions from our laws might seem an admirable thing for social choice theorists to do, but why should those who accept Arrow's theorem look for contradictions in actual democracies when they know from a theoretical point of view that any democracy *must* contain them? Why try to improve logically what is logically incapable of being improved?

It is an embarrassment that forty years after its first appearance the proof for the theorem is widely considered to be sound, that is, to have true premisses and be valid, and the result is believed to be an important insight into democratic decision making. Yet, as we have seen, even if Arrow's argument were cogent, he has at best demolished a democratic strawman, a scarecrow clothed in the language of mathematical logic, but whose substance is unrelated to the bones, blood and flesh of actual democracies. We can only guess (as I did in the above bibliography) what might have been if social choice theory had been built on philosophical foundations which are actually presupposed by at least some democracies. Is not it sad that social choice theorists have not rewritten *Robert's Rules of Order* and other parliamentary rule books in light of recent knowledge? Tougher problems such as rewriting city, state or federal laws and constitutions are hardly attempted. Yet, using social choice theory, we have the means to go as far beyond the

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[48] I quote from *Utah Election Laws: 1990–1991*: "If two or more persons have an equal and the highest number of votes for governor, lieutenant governor, attorney general, state auditor, state treasurer, or presidential elector, the lieutenant governor shall transmit to the governor a certified statement showing the vote cast for those persons. The tie shall be decided by lot in the presence of the governor and the lieutenant governor at a time fixed by the governor"(UEL 1990 113 [20–1–9]). The contradiction between the Constitution and the legislation was pointed out long ago. See ALP 1978 UT–5.

[49] I wish to thank Jim Green of the Arizona House of Representatives for providing me with this information.

eighteenth century in writing political laws and constitutions as we have gone beyond that century in natural science. What we lack is a daring and imaginative and persistent attempt to appropriate the vast knowledge of the twentieth century and focus it on the improvement of the actual means of social choice. In my judgment, a social choice theory which satisfies the highest standards of mathematical logic, empirical observation, and experimental science, allows the possibility, indeed the fair prospect, that we can in general set, and in many particular cases achieve, extremely rigorous standards for what constitutes "the consent of the governed" or "the deliberate sense of the people" or "the common reason of society." Democracy could then become not just the best of a bad lot of governments, as Churchill would have it, but a form of government perfectly suited to human nature, and the greatest triumph in the cause of humanity.[50]

Previous attempts at refuting Arrow's theorem have failed not because relevant or cogent objections were not raised -- in fact, as I indicated at the outset, I have used some of them -- but rather because the question of what constitutes excellence in social choice theory was not faced. All disciplines contain crackpot, poor, mediocre, and good-but-not-first-rate work. Yet it is the quality of what the practitioners of the discipline consider achievements deserving

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[50] An explication and defense of this point of view can be found in DeLong 1992(?). The vision that it is possible to create a government perfectly in harmony with human nature is, of course, that of the Founding Fathers. George Washington, for example, declared the purpose of the American revolution was to achieve "the last stage of perfection to which human nature is capable of attaining"(1931 26:227). Washington's interests were both ideal, as expressed here, and practical, as expressed in his leadership in that revolution and in the founding of the American government. Jefferson's interests were likewise both ideal and practical. The ideal political system described in the Declaration is an example of the former, whereas he was even more interested in the practical problem of constructing actual governments. And for these governments he harbored perfectionist aspirations: ". . . I dare say that in time all these [State governments] as well as their central government, like the planets revolving round their common sun, acting and acted upon according to their respective weights and distances, will produce that beautiful equilibrium on which our Constitution is founded, and which I believe it will exhibit to the world in a degree of perfection, unexampled but in the planetary system itself"(1892 7:210). In 1790 John Jay observed, "How far [the present national government] may be still distant from the degree of perfection to which it may possibly be carried, time only can decide"(3:388). In my judgment social choice theory would be a necessary ingredient in any adequate answer to Jay's question which we might give today. This paradox that perfectionist ideals can often serve as the inspiration and foundation of the most practical human creations can be illustrated in science. Thus, Turing in trying to solve a highly theoretical problem in mathematical logic gave the first definition (in 1936) of a perfect computing machine, a conception which captures the essence of any actual computer. But Turing's interests were not limited to ideal machines for he also directed the construction of one of the world's first electronic computers.

of the highest honor that decisively influence how the discipline will be judged. Such honor can not be given freely without consequences for the esteem with which the discipline is held. Consider the contrast between mathematics, on the one hand, and theology, on the other. There have been hundreds of proposed proofs of Fermat's last theorem and, so far, every one of them is wrong. A single error in the argument and the proposed proof is dismissed. In sharp contrast, the errors of St. Thomas Aquinas in his proofs for the existence of God have been pointed out again and again but the proofs never seem to lose their place of honor -- we are told they "somehow" reveal profound truths about reality. One of the prices paid for this honor is a widespread belief among many intellectuals that such theology is worthy only of dismissal or contempt.

What about social choice theory? In mathematics, if a theorem is proposed and the argument is subsequently found to be invalid, no credit is given for the claimed result (which might, after all, be false) and none for the refutation (it being considered a housekeeping chore for which no special credit should be expected or deserved). That is what should happen here: both the theorem and all refutations -- including this one -- should be forgotten. At least that is what we should expect if high standards are maintained. Yet, from a logical point of view, what are we going to say of a discipline, which however admirable in other respects, nevertheless tolerates Arrow's using the replacement rule for equality in intensional contexts? What are we going to say of a discipline which does not dismiss a proof which ignores the logical difficulties of self-reference? Both these objections were made above. Here is a new criticism which further illustrates lax logical standards. In logic a distinction is made between an *object language* and a *metalanguage*. A metalanguage is a language in which we talk about the object language. The metalanguage does not use the expressions of the object language, but only the names of the expressions in the object language. The names are most often formed by using quotation marks around the object-language expression. A failure to pay attention to the distinction can lead to invalid arguments. For example:

Ralph lives in Hartford.
Hartford is a word.
Therefore, Ralph lives in a word.

The trouble is with the second premiss which should read:

"Hartford" is a word.

This premiss is in the metalanguage and is about an expression in English and is not about a city. The first premiss is in the object language and concerns a person and a city, not a linguistic expression. It is the mixing of the two languages which causes the fallacy.[51] Arrow also mixes languages. The principle of Nondictatorship, for example, is about the properties of individuals, and states that none may be a dictator. It is in the object language. Collective Rationality, on the other hand, is about the outcome of a choice function and is in the metalanguage. At the very least Arrow's argument would have to be reformulated in this respect to meet the standards of correct argument in mathematical logic.

Provided only that a high degree of rigor is maintained, pure mathematicians have a great deal of freedom. For example, a geometer can consider 3–dimensional or 4–dimensional or *n*–dimensional space. This can be in a geometry which is Euclidean, Lobachevskian, Riemannian or something else entirely. The social choice theorist has a similar freedom and this freedom, like that of the pure mathematician, ought to be exercised. However, when the geometer (turned applied mathematician) makes the claim that physical space has a 4–dimensional Riemannian structure, then the freedom of pure mathematics ends and the requirement of empirical evidence begins. An enormous amount of careful work has gone into the effort of verifying or refuting claims about the spacial structure of the universe. When the social choice theorist (turned applied mathematician) makes the claim that he is modeling democracy, then freedom ends and empirical work must begin. For example, one would have to specify as carefully as possible what the nonlogical and nonmathematical terms of the theory mean, and then show that the premisses are in fact true of some exactly described reality when those terms are understood in the sense specified. Without this requirement mathematical logic becomes a mere facade which gives the impression of certainty and precision but in fact obscures clear thought. Now, from an empirical point of view, what are we going to say about the methods used to establish the truth of the premisses of Arrow's theorem? As we have seen, the assumptions of the proof are contrary to such obvious and long–standing democratic devices as tie–breaking procedures,

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[51] The strict separation of language and metalanguage is one of the things that distinguishes Gödel's argument (which is valid) from the invalid Richard Paradox (to which Gödel's argument bears analogy).

the possibility of voting to change procedures in voting, the employment of self–paternalistic devices, and so forth. Yet the theorem is supposedly about democracy, not a fantasy about what democracy might be; it is supposedly grounded in a lexical, not a merely stipulative, definition. Would any empirical claim in physics be given credence if its verification were as casual as we are given for truth of the premisses of Arrow's theorem? (Or should we opt for social choice theology and conclude that, in spite of logical and empirical lapses, the theorem "somehow" reveals an important truth about democracy?) Nor can we say that Arrow's formulation is a legitimate simplification unless we have evidence that the contradiction was not introduced by the simplification process itself.[52] It might be claimed that actual democracies are too complex for social choice theory. Well, perhaps a mountain is too complex an object for Euclidean geometry but the solution is not to abandon the attempt to understand mountains but to devise a fractal geometry which does give insight into such objects. In like manner, we must devise a social choice theory adequate for actual democracies.[53] To do so we would have to study carefully the history of "democracy" and related words in English and see how they came to have their respective meanings. Here we would attempt to use the most rigorous standards and the most meticulous procedures which philology and the history of ideas have developed. Indeed the availability of computer technology and data bases for the relevant documents would, perhaps, enable us to exceed the standards which have already been achieved using older methods. Building a theory of social choice on such careful work would make silly, I believe, such claims as "there are as many meanings of *democracy* as there

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[52] Note that the statement "It may be chartreuse but it is definitely not celadon" becomes contradictory if we simplify by replacing "chartreuse" and "celadon" by "green". Such a replacement might be innocuous in many contexts but not in this one. *Mutatis mutandis*, the same objection applies to Arrow's conditions on constitutions: they must be shown to be true of democratic practices and not be introduced by a false assumptions about them. Otherwise his comment (1967 15) that the four conditions are sufficient for a contradiction loses its bite.

[53] For example, suppose the results of an election are 1,479,843 for candidate A and 1,479,852 for candidate B. In most systems of social choice, candidate B will have won. In any actual case, however, such a result would probably lead to a contested election since there would be legitimate doubt as to who had won. How close must elections be before it is reasonable to contest them? What are the best procedures for dealing with the disagreement? How might one so structure the procedures of voting to reduce the likelihood of contested elections? Such questions would all be part of a social choice theory which aimed to be adequate for actual democracies.

are democrats" or "*democracy* can mean anything you want it to mean." In addition, we would need experiments with speakers of English as to the meanings which they assign to the word, observe how election officials do their work, including what they do when they have ties and contested elections, and so forth. All this would not be easy. The creation of the data bases alone would involve millions of hours of work. But these high standards would be repaid in the certainty of the results, the increased respect of the discipline and the applicability of the theory to actual democracies. My aim has been to show not only that Arrow's work falls short of demonstrating the theorem but, in addition, that there are many reasons for thinking that there will not be a significant theorem -- which could justifiably be called Arrow's theorem -- in any social choice writing which achieves the highest standards of excellence now possible with regard to mathematical logic, empirical observation, linguistic analysis and political experiment.[54] In short, democracies and democratic theory will face many challenges and difficulties in the future but Arrow's theorem will not be one of them.

It will perhaps be objected that these standards are too exacting. One could quote Aristotle's statement that "it is the mark of an educated man to look for precision in each class of things just so far as the nature of the subject admits"(1984 2:1730 [1094b 24–5]). Yet how much precision does social choice theory allow? We must be cognizant of the fact that, for example, biology has achieved much more precision than Aristotle believed the subject admitted. One can only answer the question of precision by vigorously and imaginatively pushing attempts at it until bounds become evident. Even then, one should recognize that those bounds may be only temporary. Aristotle's dictum should help us avoid a false exactitude, not serve as a balm for the lazy and unimaginative, nor as an excuse to neglect further attempts at greater precision than has hitherto been possible.

It might also be objected that these standards are too expensive. But we cannot assume that it is much easier and cheaper to understand social choice than it is to understand, say, the atom or the human genome. For example, to

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[54] I have tried to be as careful as I could in this essay but I certainly do not claim to meet these standards. (Nor is it necessary in a refutation, which can contain numerous errors but succeeds if somewhere it establishes that one or more premisses of a supposed result are false, or that its argument is invalid.) In any case, there would be very little work achieving such standards but, what little would exist, would deserve the highest honor.

understand social choice in an actual political entity, would one not have to create a dynamic model which was continuously updated? For the United States the difficulty and expense of doing this might be as much as or more than running the weather service or mapping the human genome. We have no right to assume that knowledge of the political world using social choice theory will be any cheaper than knowledge of the physical or biological world.

"Why did you bother to write this refutation," I can imagine a reader thinking, "for if you are wrong, it will justly cause you embarrassment, but if you are right it will, by you own admission, deserve to be forgotten?" The answer is dependent on my view that social choice theory is potentially as important for society as is the theory of computation. I have been extremely critical of Arrow's proof but I greatly applaud the idea -- for which Arrow deserves perhaps the most credit -- of showing that social choice is capable of being understood through the medium of a powerful symbolism. I also believe that there is much in contemporary social choice theory that meets high standards of logical excellence. But that logic when applied must be shown to be grounded in empirical reality. All over the world democracies are struggling, and social choice theory has contributed little to help them. Would the voting schemes actually adopted in the world's new democracies be judged by most social choice theorists as being the best possible (even given practical constraints)? Again, imagine the President is confronted with some important social issue -- such as drugs or the savings and loan crisis -- and an aide says "Let us ask the social choice theorists what they think?" If those theorists accepted Arrow's characterization of democracy in his social choice writings, would not the President be justified in saying: "Be serious!" American democracy may be in as desperate a condition in the twentieth century as it was under the Articles of Confederation in the eighteenth. The media correctly report a steady stream of incompetence, ineptitude, corruption, and political gridlock. To many Americans Washington and the President seem as distant and unresponsive now as London and George III did in revolutionary times. The vulnerable of America cry "Help! Help!" and there is little help. And social choice theory, which could help in these matters, is at present impotent. But it need not be, provided various impediments are removed. In my judgment one of the most important of those impediments is the widespread acceptance in social choice literature of both the standards of proof and the philosophical presuppositions implicit in Arrow's theorem. This is why I believe a refutation is the worth the bother.

It is said that the Mayans knew of the wheel but used it only for toys and not for the practical problems they faced in moving heavy objects. Should we marvel why they did not use it to reduce unnecessary hardship, when we ourselves possess a social choice theory, which has too often remained a plaything for intellectuals, instead of becoming a practical technology to improve our lives? When we think of the opportunities that have been squandered, there is a strong objection to allowing Arrow's theorem to retain the vast prestige it has. Social choice theorists should ask: What is being done in social choice theory to help achieve the great moral goals of democracy: dignity, equality, freedom, and the flourishing of every human being? What standards are required for a proof, for a refutation, or for a work deserving the highest honor? What are the best social choices with regard to the future development of social choice theory?

References

Note: If a particular work has no listed author or editor, then it is referred to by an abbreviation taken from the first letters of words in the title. It is alphabetically placed below as if that abbreviation were the name of the author. Also if a citation made above is to a multivolumed work only the date of the first volume is given but the full range of dates is furnished below.

Adams, John

1794 *A Defence of the Constitutions of Government of the United States of America.* 3 volumes. London: John Stockdale.

ALP

1978 *An Analysis of Laws and Procedures Governing Contested Elections and Recounts: Final Report.* 3 volumes. Washington: National Clearinghouse on Election Administration.

Anderson, Alan Ross and Nuel D. Belnap, Jr.

1975 *Entailment: the Logic of Relevance and Necessity.* Volume 1. Princeton: Princeton University Press.

Aristotle

1984 *The Complete Works of Aristotle.* Edited by Jonathan Barnes. 2 volumes. Princeton: Princeton University Press.

Arrow, Kenneth J.

1950 "A Difficulty in the Concept of Social Welfare." *Journal of Political Economy* 58 (August): 328–346.

1951 "Mathematical Models in the Social Sciences." In *The Policy Sciences: Recent Developments in Scope and Method*, edited by

Daniel Lerner and Harold D. Lasswell, 129–155. Standford: Standford University Press.

1963 *Social Choice and Individual Values*. Second edition. New Haven: Yale University Press.

1967 "Public and Private Values." In *Human Values and Economic Policy*, edited by Sidney Hook, 3–21. New York: New York University Press.

1967a "Values and Collective Decision–Making." In *Philosophy, Politics, and Society*, Third Series, edited by Peter Laslett and W. G. Runciman, 215–32. Oxford: Basil Blackwell.

1969 "The Organization of Economic Activity: Issues Pertinent to the Choice of Market Versus Nonmarket Allocation." In *The Analysis and Evaluation of Public Expenditures: The PPB System*, volume 1, Joint Economic Committee, Congress of the United States, 47–64. Washington, D. C.: U. S. Government Printing Office.

1973 "Formal Theories of Social Welfare." In *Dictionary of the History of Ideas*, 5 volumes, edited by Philip P. Wiener, 4:276–284. New York: Charles Scribner's Sons.

1973a "Social Responsibility and Economic Efficiency." *Public Policy* 21 (Summer): 303–17.

1973b "Some Ordinalist–Utilitarian Notes on Rawls's Theory of Justice." *Journal of Philosophy* 70 (May): 245–63.

1974 "A Case for Redistributing Income: Taxation and Democratic Values." *The New Republic* 171 (November 2): 23–25.

1974a "General Economic Equilibrium: Purpose, Analytic Techniques, Collective Choice." *American Economic Review* 64 (June): 253–272.

1977 "Current Developments in the Theory of Social Choice." *Social Research* 44 (Winter): 607–622.

1983–85 *Collected Papers of Kenneth J. Arrow.* 6 volumes. Cambridge: Harvard University Press.

1985 "Distributive Justice and Desirable Ends of Economic Activity." In *Issues in Contemporary Macroeconomics and Distribution*, edited by George F. Feiwel, 134–156. Albany: State University of New York Press.

1987 "Arrow's Theorem." In *The New Palgrave: A Dictionary of Economics*, 4 volumes, edited by John Eatwell, Murry Milgate, and Peter Newman, 1:124–126. London: Macmillan.

Bartlett, Steven J. and Peter Suber

1987 *Self–Reference: Reflections on Reflexivity.* Dordrecht: Martinus Nijhoff.

Bentham, Jeremy

1859 *The Works of Jeremy Bentham.* Edited by John Bowring. 11 volumes. London: Simpkin, Marshall.

1968 *The Correspondence of Jeremy Bentham.* Edited by Timothy L. S. Sprigge. Volume 1. London: Athlone Press.

Bergson, Abram

1954 "On the Concept of Social Welfare." *Quarterly Journal of Economics* 68 (May): 233–52.

Blyth, Colin R.

1972 "Some Probability Paradoxes in Choice from Among Random Alternatives." *Journal of the American Statistical Association* 67 (June): 366–73.

Brans, Steven J. and Peter C. Fishburn
1983 *Approval Voting*. Boston: Birkhäuser.

BWTSS
1959 "Bartlett Wins Toss for Senior Senator." *The New York Times* (January 7): 25.

Churchill, Winston S.
1974 *Winston S. Churchill, His Complete Speeches: 1897–1963*. Edited by Robert R. James. 8 volumes. New York: Chelsea House.

Cicero
1935 *The Speeches*. Edited by N. H. Watts. Cambridge: Harvard University Press.

Copi, Irving M.
1986 *Introduction to Logic*. Seventh edition. New York: Macmillan and Company.

DeLong, Howard
1992(?) *The Court of Common Reason: The American Revolution and the Aspiration for Democratic Ideals*. Forthcoming.

Demeter, George
1969 *Demeter's Manual of Parliamentary Law and Procedure*. Boston: Little, Brown and Co..

DRCPH
1978 "Democrats Retain Control of Pennsylvania House." *The New York Times* (November 22): 24.

Feiwel, George R., editor.
1987 *Arrow and the Ascent of Modern Economic Theory*. New York: New York University Press.

1987a *Arrow and the Foundations of the Theory of Economic Policy.* New York: New York University Press.

Friedland, Edward I. and Stephen J. Cimbala

1973 "Process and Paradox: the Significance of Arrow's Theorem." *Theory and Decision* 4 (Summer): 51–62.

Gardner, Martin

1970 "The Paradox of the Nontransitive Dice and the Elusive Principle of Indifference." *Scientific American* 223 (December): 110–14.

1974 "On the Paradoxical Situations that Arise from Nontransitive Relations." *Scientific American* 231 (October): 120–25.

1976 "On the Fabric of Inductive Logic, and Some Probability Paradoxes." *Scientific American* 234 (March): 119–124.

Gardner, Richard J. and Stan Wagon

1989 "At Long Last, the Circle Has Been Squared." *Notices of the American Mathematical Society*, 36 (December): 1338–43.

Gellius, Aulus

1927 *The Attic Nights of Aulus Gellius.* Edited by John C. Rolfe. 3 volumes. New York: G. P. Putnam's Sons.

Hall, Ursula

1964 "Voting Procedure in Roman Assemblies." *Historia* 13 (Number 3): 267–306.

Hamilton, Alexander

1961–87 *The Papers of Alexander Hamilton.* Edited by Harold C. Syrett. 27 volumes. New York: Columbia University Press.

Hansen, Mogens Herman

1987 *The Athenian Assembly in the Age of Demosthenes.* Oxford: Basil Blackwell.

1989 *Was Athens a Democracy?* Copenhagen: The Royal Danish Academy of Sciences and Letters.

Haraszti, Zoltan
1952 *John Adams & The Prophets of Progress.* Cambridge: Harvard University Press.

Heliodorus, of Emesa
1961 *Ethiopian Story.* Edited by Walter Lamb. New York: E. P. Dutton.

Herzberg, Roberta
1986 "McCloskey versus McIntyre: Implications of Contested Elections in a Federal Democracy." *Publius* 16 (Summer): 93–109.

Hofstadter, Douglas R.
1982 "About Nomic: a Heroic Game that Explores the Reflexivity of the Law." *Scientific American* 246 (June): 16–28.

Holford–Strevens, Leofranc
1988 *Aulus Gellius.* Chapel Hill: University of North Carolina Press.

Jay, John
1890–93 *The Correspondence and Public Papers of John Jay.* Edited by Henry Phelps Johnston. 4 volumes. New York: G. P. Putnam's Sons.

Jefferson, Thomas
1972 *Notes on the State of Virginia.* New York: W. W. Norton & Company.

1892–99 *The Works of Thomas Jefferson.* Edited by Paul Leicester Ford. 10 volumes. New York: G. P. Putnam's Sons.

1903–4 *The Writings of Thomas Jefferson.* Edited by Andrew A. Lipscomb. Washington: Thomas Jefferson Memorial Association.

JHRUS

1821 *Journal of the House of Representatives of the United States.* Washington: Gales and Seaton: 364–371.

JVS

1891 "Johnston *et. al.* v. State *ex. rel.* Sefton." *Northeastern Reporter* 27: 422–3.

Kleene, Stephen Cole

1967 *Mathematical Logic.* New York: John Wiley & Sons.

Larsen, J. A. O.

1949 "The Origin and Significance of the Counting of Votes." *Classical Philology* 44 (July): 164–181.

Libanius

1903–27 *Libanii Opera.* Edited by Richard Foerster. 12 volumes. Leipzig: B. G. Teubner.

Locke, John

1988 *Two Treatises of Government.* Student edition. Edited by Peter Laslett. Cambridge: Cambridge University Press.

Lowrie, Walter and Walter S. Franklin, editors.

1834 *American State Papers: Documents, Legislative and Executive, of the Congress of the United States.* Volume 2. Washington: Gales and Seaton.

Madison, James

1962–89 *The Papers of James Madison.* Edited by William Thomas Hutchinson et. al. 16 volumes. Chicago: University of Chicago Press (volumes 1–10) and Charlottesville: University Press of Virginia (volumes 11–16).

Mill, John Stuart

1948 *Utilitarianism, Liberty, and Representative Government.* New York: E.P. Dutton & Co., Inc..

1978 *On Liberty.* Edited by Elizabeth Rapaport. Indianapolis: Hackett Publishing Company.

Monro, D. H.

1967 "Jeremy Bentham." In *The Encyclopedia of Philosophy*, edited by Paul Edwards, 8 volumes, 1:280–85. New York: Macmillan Company & Free Press.

Peirce, Charles Sanders

1933–58 *Collected Papers of Charles Sanders Peirce.* Edited by Charles Hartshorne, Paul Weiss, and Arthur W. Burks. 8 volumes. Cambridge: Harvard University Press.

Plato

1980 *The Laws of Plato.* Edited by Thomas L. Pangle. New York: Basic Books.

1985 *The Republic.* Edited by Richard W. Sterling and William C. Scott. New York: W. W. Norton.

Pliny, the Younger

1940 *Letters.* Edited by W. M. L. Hutchinson. 2 volumes. New York: Macmillan.

Quine, Willard Van Orman

1966 *The Ways of Paradox and Other Essays.* New York: Random House.

Quintilian

1984 *The Minor Declamations Ascribed to Quintilian.* Edited by Michael Winterbottom. Berlin: Walter de Gruyter.

Rescher, Nicholas

1969 *Essays in Philosophical Analysis.* Pittsburgh: University of Pittsburgh Press.

RLM

1863 *Rhetores Latini Minores.* Edited by Carolus Halm. Leipzig: B. G. Teubner.

Russell, D. A.

1983 *Greek Declamation.* Cambridge: Cambridge University Press.

Samuelson, Paul

1966–86 *The Collected Scientific Papers of Paul A Samuelson.* 5 volumes. Cambridge: MIT Press.

Sen, Amartya

1982 *Choice, Welfare and Measurement.* Cambridge: MIT Press.

1987 "Social Choice." In *The New Palgrave: A Dictionary of Economics*, 4 volumes, edited by John Eatwell, Murry Milgate, and Peter Newman, 4:382–393. London: Macmillan.

Smith, Adam

1982 *The Theory of Moral Sentiments.* Indianapolis: Liberty Classics.

Staveley, E. S.

1972 *Greek and Roman Voting and Elections.* Ithaca: Cornell University Press.

Suber, Peter

1990 *The Paradox of Self-Amendment: a Study of Logic, Law, Omnipotence, and Change.* New York: Peter Lang.

Taylor, Lily Ross

1966 *Roman Voting Assemblies.* Ann Arbor: The University of Michigan Press.

Trussell, C. P.

1959 "Congress Greets Alaska Members." *The New York Times* (January 8): 12.

1959a "Congress Picks 3 New Members From 50th State." *The New York Times* (August 25): 1, 15.

Tullock, Gordon

1979 "Public Choice in Practice." In *Collective Decision Making: Applications from Public Choice Theory*, edited by Clifford S. Russell, 27–45. Baltimore: Johns Hopkins University Press.

Turing, Alan M.

1936–7 "On Computable Numbers, With an Application to the Entscheidungsproblem." *Proceedings of the London Mathematical Society*, Series 2, 42: 230–65.

1950 "Computing Machinery and Intelligence." *Mind* 59 (October): 433–60.

Tversky, Amos

1969 "Intransitivity of Preferences." *Psychological Review* 76 (January): 31–48.

TVRO

1978 "Tie–Vote Recount Ordered." *The New York Times* (November 11): 8.

UEL

1990 *Utah Election Laws: 1990–1991*. Prepared under the direction of W. Val Oveson. Provo: Code Company.

van den Haag, Ernest

1967 "Normative and Analytical Welfare Economics: Arrow's Pareto Principle." In *Human Values and Economic Policy*, edited by Sidney Hook, 181–92. New York: New York University Press.

von Neumann, John

1959 "On the Theory of Games of Strategy." In *Contributions to the Theory of Games*, volume 4, edited by A.W. Tucker and R.D. Luce, 13–42. Princeton: Princeton University Press.

von Neumann, John and Oskar Morgenstern

1967 *Theory of Games and Economic Behavior*. New York: John Wiley & Sons.

Waley, Daniel

1969 *The Italian City–Republics*. New York: McGraw–Hill Book Company.

Washington, George

1931–44 *The Writings of George Washington*. Edited by John Clement Fitzpatrick. 39 volumes. Washington: United States Government Printing Office.

Wolff, Robert Paul

1970 *In Defense of Anarchism*. New York: Harper & Row.

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